Parallel Computation/Program Issues

- Dependency Analysis:
  - Types of dependency
  - Dependency Graphs
  - Bernstein's Conditions of Parallelism
- Asymptotic Notations for Algorithm Complexity Analysis
- Parallel Random-Access Machine (PRAM)
  - Example: sum algorithm on P processor PRAM
- Network Model of Message-Passing Multicomputers
  - Example: Asynchronous Matrix Vector Product on a Ring
- Levels of Parallelism in Program Execution
- Hardware Vs. Software Parallelism
- Parallel Task Grain Size
- Software Parallelism Types: Data Vs. Functional Parallelism
- Example Motivating Problem With high levels of concurrency
- Limited Parallel Program Concurrency: Amdahl's Law
- Parallel Performance Metrics: Degree of Parallelism (DOP)
  - Concurrency Profile

+ Average Parallelism

- Steps in Creating a Parallel Program:
  - 1- Decomposition, 2- Assignment, 3- Orchestration, 4- (Mapping + Scheduling)
  - Program Partitioning Example (handout)
  - Static Multiprocessor Scheduling Example (handout)

PCA Chapter 2.1, 2.2

Parallel Programs: Definitions

- A parallel program is comprised of a number of <u>tasks</u> running as threads (or processes) on a number of processing elements that cooperate/communicate as part of a single parallel computation.

The processor with max. execution time determines parallel execution time

Parallel Execution Time

Computation	Communication	Other Parallelization Overheads

- ***Task*:**
  - Arbitrary piece of undecomposed work in parallel computation
  - Executed sequentially on a single processor; concurrency in parallel computation is only across tasks.

i.e At Thread Level Parallelism (TLP)

- ***Parallel or Independent Tasks:***
  - Tasks that with <u>no dependencies</u> among them and thus can run in parallel on different processing elements.
- ***Parallel Task Grain Size:*** The amount of computations in a task.
- ***Process (thread)*:**
  - Abstract program entity that performs the computations assigned to a task
  - Processes communicate and synchronize to perform their tasks
- ***Processor or (Processing Element)*:**
  - Physical computing engine on which a process executes sequentially
  - Processes virtualize machine to programmer
    - First write program in terms of processes, then <u>map</u> to processors
- **<u>Communication to Computation Ratio (C-to-C Ratio):</u>** Represents the amount of resulting communication between tasks of a parallel program

In general, for a parallel computation, a lower C-to-C ratio is desirable and usually indicates better parallel performance

Factors Affecting Parallel System Performance

Slide 29 from Lecture 1 Repeated

- **Parallel Algorithm Related:**
  - Available concurrency and profile, grain size, uniformity, patterns.
    - Dependencies between computations represented by dependency graph
  - Type of parallelism present: Functional and/or data parallelism.
  - Required communication/synchronization, uniformity and patterns.
  - Data size requirements.
  - Communication to computation ratio (C-to-C ratio, lower is better).

i.e Inherent Parallelism

- **Parallel program Related:**
  - Programming model used.
  - Resulting data/code memory requirements, locality and working set characteristics.
  - Parallel task grain size.
  - Assignment (mapping) of tasks to processors: Dynamic or static.
  - Cost of communication/synchronization primitives.

Results in parallelization overheads/extra work

- **Hardware/Architecture related:**
  - Total CPU computational power available.
  - Types of computation modes supported.
  - Shared address space Vs. message passing.
  - Communication network characteristics (topology, bandwidth, latency)
  - Memory hierarchy properties.

+ Number of processors (hardware parallelism)

Impacts time/cost of communication

Concurrency = Parallelism

Dependency Analysis & Conditions of Parallelism

- **Dependency analysis is concerned with detecting the presence and type of dependency between tasks that prevent tasks from being independent and from running in parallel on different processors and can be applied to tasks of any grain size.*** **Down to task = instruction**
  - **Represented graphically as task dependency graphs.**
- **Dependencies between tasks can be 1- algorithm/program related or 2- hardware resource/architecture related.**

1 **Algorithm/program Task Dependencies:**

- **Data Dependence:**
  - **True Data or Flow Dependence** — Algorithm Related
  - **Name Dependence:** — Parallel Program and Programming Model Related
    - **Anti-dependence**
    - **Output (or write) dependence**
- **Control Dependence** — Algorithm Related

2 **Hardware/Architecture Resource Dependence**

Parallel architecture related

***** **Task Grain Size:** Amount of computation in a task

A task only executes on one processor to which it has been mapped or allocated

Conditions of Parallelism: Data & Name Dependence

S1
..
..
S2

Program Order

As part of the algorithm/computation

Assume task S2 follows task S1 in sequential program order

1 **True Data or Flow Dependence:** Task S2 is data dependent on task S1 if an execution path exists from S1 to S2 and if at least one output variable of S1 feeds in as an input operand used by S2

Represented by S1 ⟶ S2 in task dependency graphs

Name Dependencies

2 **Anti-dependence:** Task S2 is antidependent on S1, if S2 follows S1 in program order and if the output of S2 overlaps the input of S1

Represented by S1 ⟶ S2 in dependency graphs

3 **Output dependence:** Two tasks S1, S2 are output dependent if they produce the same output variables (or output overlaps)

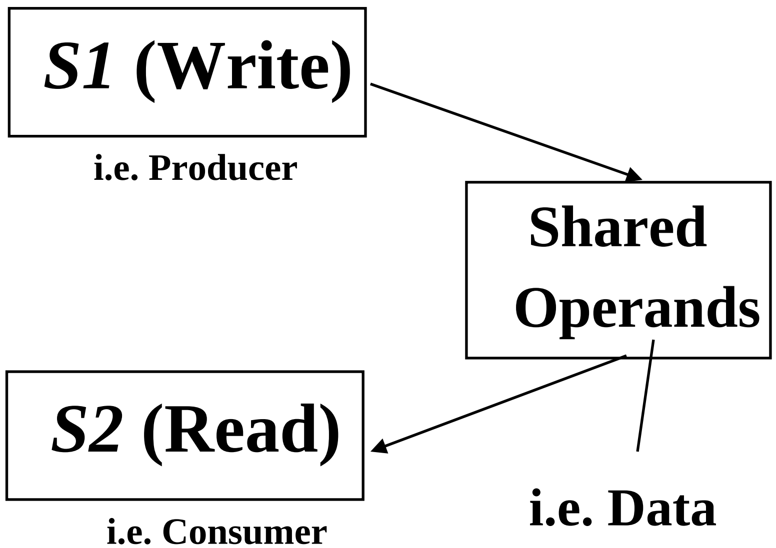
Represented by S1 ⟶ S2 in task dependency graphs

(True) Data (or Flow) Dependence

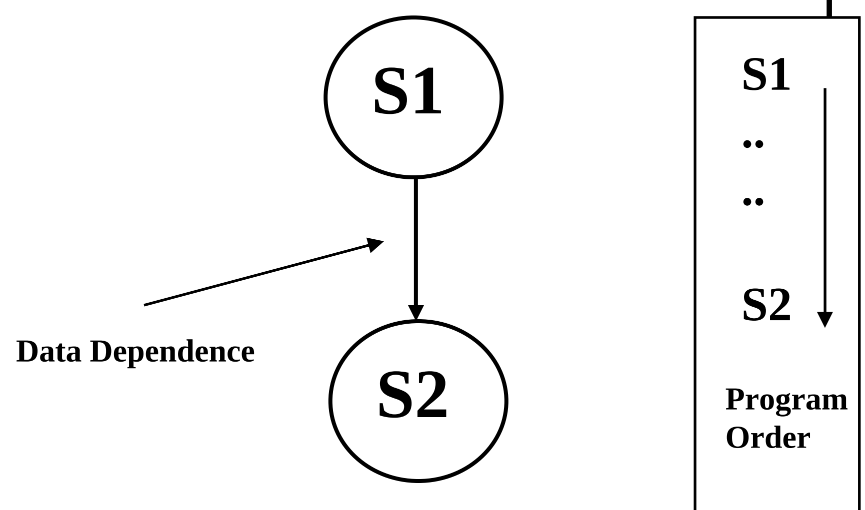
- **Assume task S2 follows task S1 in sequential program order**
- **Task S1 produces one or more results used by task S2,**
  - Then task S2 is said to be data dependent on task S1

S1 ⟶ S2

- **Changing the relative execution order of tasks S1, S2 in the parallel program violates this data dependence and results in incorrect execution.**



Task Dependency Graph Representation



Task S2 is data dependent on task S1

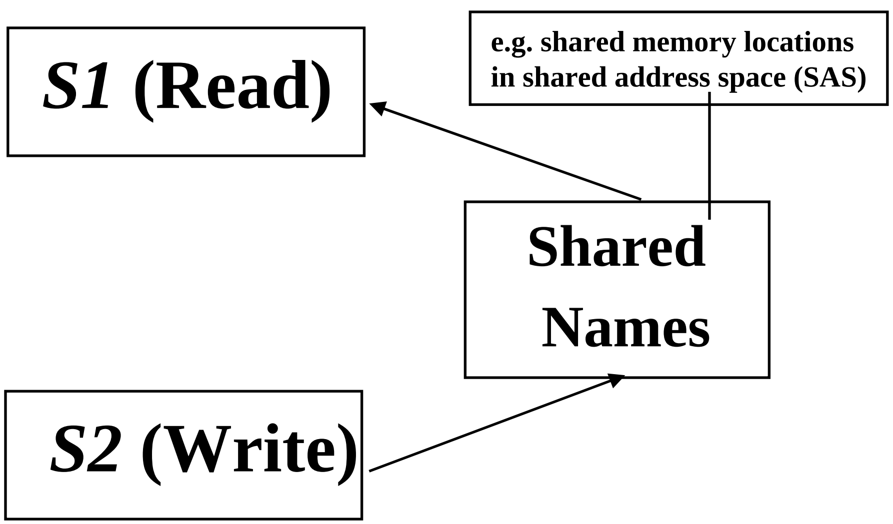
Data dependencies between tasks originate from the parallel algorithm

Name Dependence Classification: Anti-Dependence

Program Related

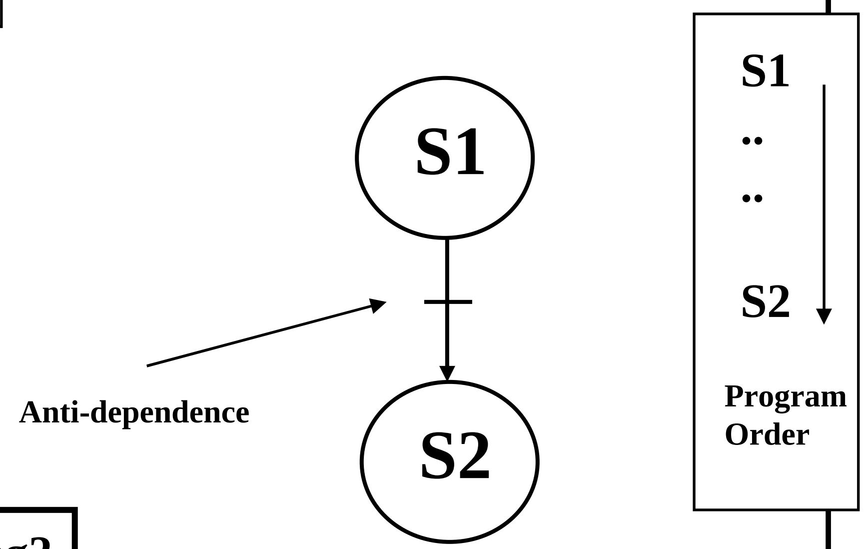
- Assume task S2 follows task S1 in sequential program order
- Task S1 reads one or more values from one or more names (registers or memory locations)
- Task S2 writes one or more values to the same names (same registers or memory locations read by S1)
  - Then task S2 is said to be anti-dependent on task S1
- Changing the relative execution order of tasks S1, S2 in the parallel program violates this name dependence and may result in incorrect execution.

S1 ⟶ S2



Task S2 is anti-dependent on task S1

Task Dependency Graph Representation



Does anti-dependence matter for message passing?

Name: Register or Memory Location

Name Dependence Classification:

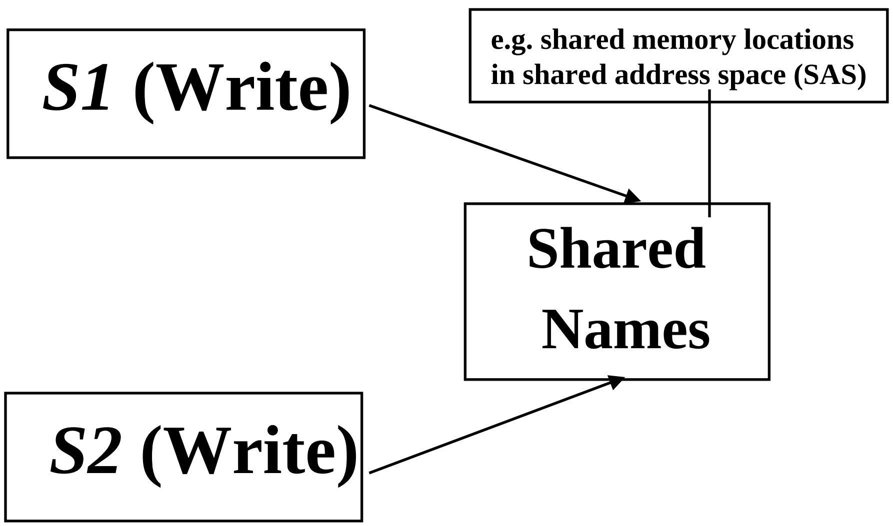
Output (or Write) Dependence

Program Related

- **Assume task S2 follows task S1 in sequential program order**
- **Both tasks S1, S2 write to the same a name or names (same <u>registers</u> or <u>memory locations</u>)**
  - Then task S2 is said to be output-dependent on task S1

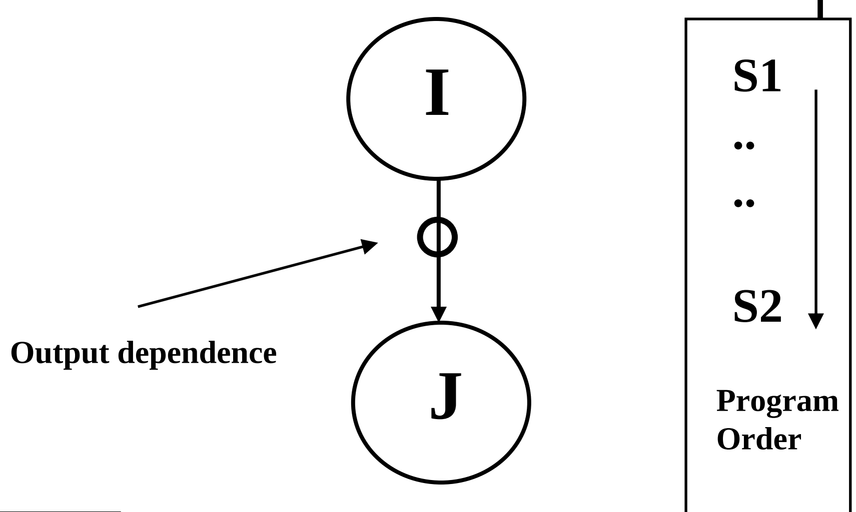
S1 ⟶o⟶ S2

- **Changing the relative execution order of tasks S1, S2 in the parallel program violates this name dependence and may result in incorrect execution.**



Task S2 is output-dependent on task S1

Task Dependency Graph Representation



Does output dependence matter for message passing?

<u>Name</u>: Register or Memory Location

Dependency Graph Example

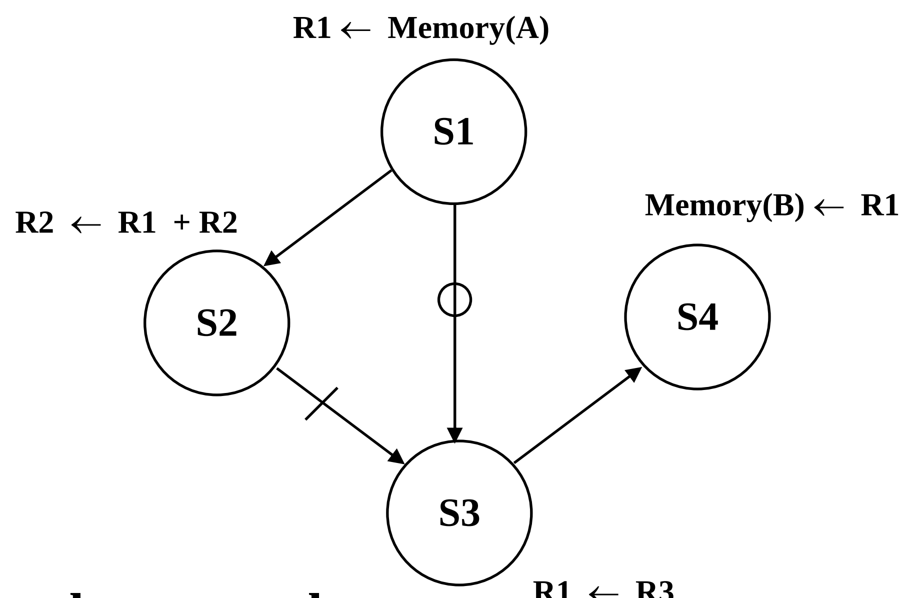
<u>Here assume each instruction is treated as a task:</u>

S1:	**Load R1, A**	**/ R1 $\leftarrow$ Memory(A) /**
S2:	**Add R2, R1**	**/ R2 $\leftarrow$ R1 + R2 /**
S3:	**Move R1, R3**	**/ R1 $\leftarrow$ R3 /**
S4:	**Store B, R1**	**/Memory(B) $\leftarrow$ R1 /**

True Date Dependence:
(S1, S2) (S3, S4)
i.e. S1 $\longrightarrow$ S2
S3 $\longrightarrow$ S4

Output Dependence:
(S1, S3)
i.e. S1 $\longrightarrow$ S3

Anti-dependence:
(S2, S3)
i.e. S2 $\longrightarrow$ S3

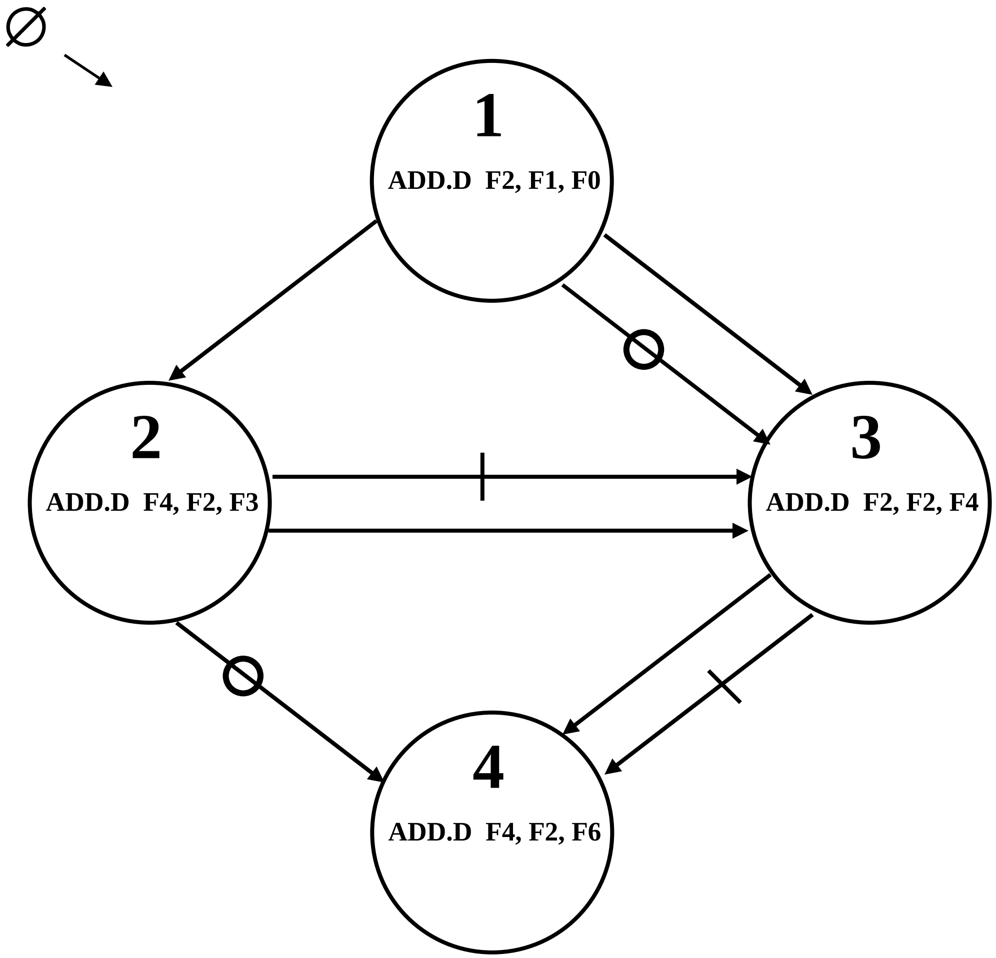


Dependency graph

Dependency Graph Example

Here assume each instruction is treated as a task

Task Dependency graph



MIPS Code

1	ADD.D	F2, F1, F0
2	ADD.D	F4, F2, F3
3	ADD.D	F2, F2, F4
4	ADD.D	F4, F2, F6

True Date Dependence:

(1, 2) (1, 3) (2, 3) (3, 4)

i.e. $1 \longrightarrow 2 \quad 1 \longrightarrow 3$
$2 \longrightarrow 3 \quad 3 \longrightarrow 4$

Output Dependence:

(1, 3) (2, 4)

i.e. $1 \longrightarrow 3 \quad 2 \longrightarrow 4$

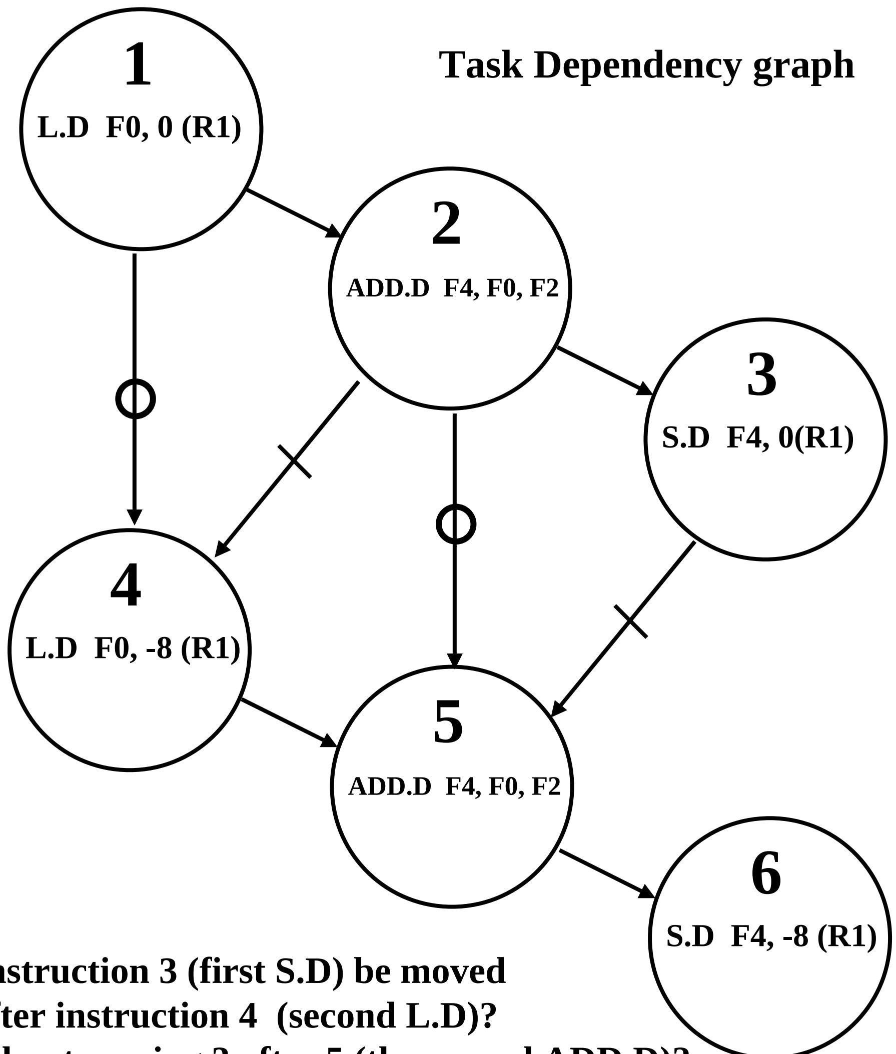
Anti-dependence:

(2, 3) (3, 4)

i.e. $2 \longrightarrow 3 \quad 3 \longrightarrow 4$

Dependency Graph Example

<u>**Here assume each instruction is treated as a task**</u>



MIPS Code

1	L.D	F0, 0 (R1)
2	ADD.D	F4, F0, F2
3	S.D	F4, 0(R1)
4	L.D	F0, -8(R1)
5	ADD.D	F4, F0, F2
6	S.D	F4, -8(R1)

True Date Dependence:
(1, 2) (2, 3) (4, 5) (5, 6)
i.e. $1 \longrightarrow 2 \quad 1 \longrightarrow 3$
$4 \longrightarrow 5 \quad 5 \longrightarrow 6$

Output Dependence:
(1, 4) (2, 5)
i.e. $1 \rightarrow 4 \quad 2 \rightarrow 5$

Anti-dependence:
(2, 4) (3, 5)
i.e. $2 \rightarrow 4 \quad 3 \rightarrow 5$

Can instruction 3 (first S.D) be moved just after instruction 4 (second L.D)? How about moving 3 after 5 (the second ADD.D)? If not what dependencies are violated?

Can instruction 4 (second L.D) be moved just after instruction 1 (first L.D)? If not what dependencies are violated?

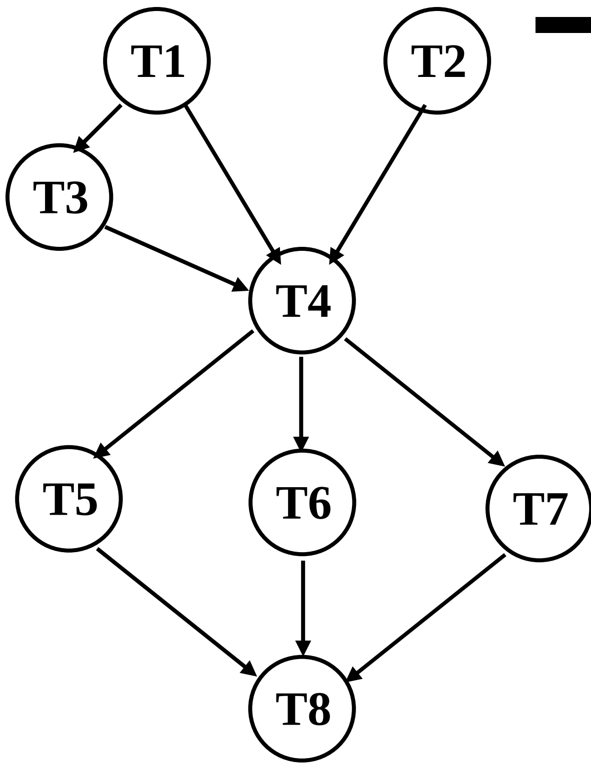
(From 551)

Example Parallel Constructs: Co-begin, Co-end

- **A number of generic parallel constructs can be used to specify or represent parallelism in parallel computations or code including (Co-begin, Co-end).**
- **Statements or tasks can be run in parallel if they are declared in same block of (Co-begin, Co-end) pair.**
- **<u>Example:</u> Given the the following task dependency graph of a computation with eight tasks T1-T8:**

<u>Possible</u> Sequential Code Order:

T1
T2
T3
T4
T5,
T6,
T7
T8



Task Dependency Graph

Parallel code using Co-begin, Co-end:

```
Co-Begin
   T1, T2
Co-End
T3
T4
Co-Begin
   T5, T6, T7
Co-End
T8
```

Conditions of Parallelism

- **Control Dependence:**
  - Order of execution cannot be determined before runtime due to conditional statements.
- **Resource Dependence:**
  - Concerned with conflicts in using shared resources among parallel tasks, including:
    - Functional units (integer, floating point), memory areas, communication links etc.
- **Bernstein's Conditions of Parallelism:**

i.e. Results produced

Two processes P_1 , P_2 with input sets I_1, I_2 and output sets O_1, O_2 can execute in parallel (denoted by $P_1 \parallel P_2$) if:

Order of P_1 , P_2 ?

$$I_1 \cap O_2 = \emptyset$$

$$I_2 \cap O_1 = \emptyset$$

$$O_1 \cap O_2 = \emptyset$$

i.e no flow (data) dependence or anti-dependence (which is which?)

i.e no output dependence

Bernstein's Conditions: An Example

- For the following instructions P_1, P_2, P_3, P_4, P_5 :
  - Each instruction requires one step to execute
  - Two adders are available

P1
Co-Begin
P1, P3, P5
Co-End
P4

P_1 : C = D x E

P_2 : M = G + C

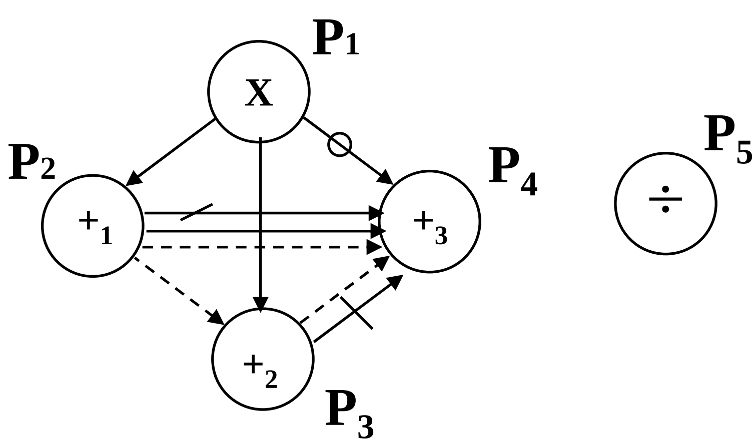
P_3 : A = B + C

P_4 : C = L + M

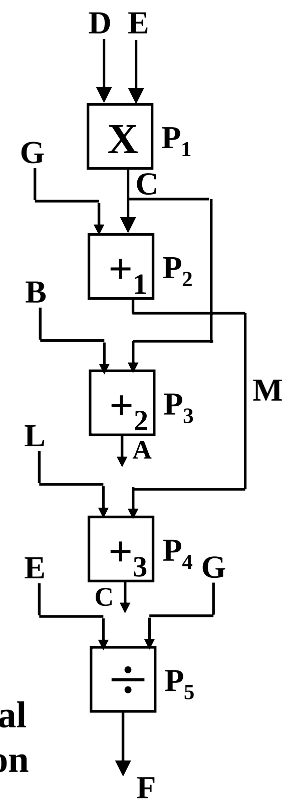
P_5 : F = G ÷ E

Using Bernstein's Conditions after checking statement pairs:

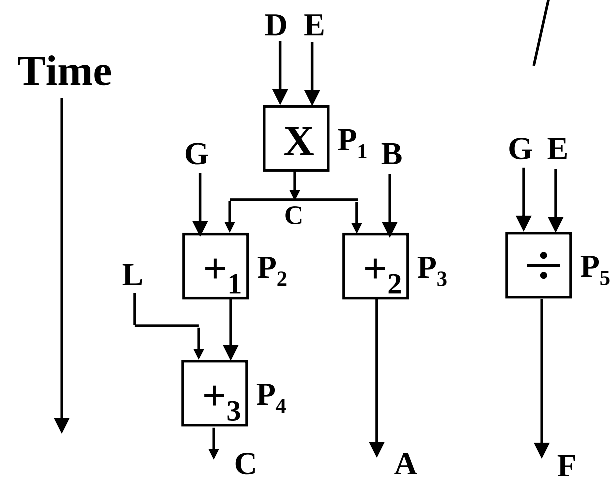
$P_1 \parallel P_5$, $P_2 \parallel P_3$, $P_2 \parallel P_5$, $P_3 \parallel P_5$, $P_4 \parallel P_5$



Dependence graph:
Data dependence (solid lines)
Resource dependence (dashed lines)



Sequential execution



Parallel execution in three steps assuming two adders are available per step

Asymptotic Notations for Algorithm Analysis

- Asymptotic analysis of computing time (computational) complexity of an algorithm T(n)= f(n) ignores constant execution factors and concentrates on:
  - Determining the order of magnitude of algorithm performance.
  - How quickly does the running time (computational complexity) grow as a function of the input size.

Rate of growth of computational function

- We can compare algorithms based on their asymptotic behavior and select the one with lowest rate of growth of complexity in terms of input size or problem size n independent of the computer hardware.

♦ **Upper bound: Order Notation (Big Oh)** O( )

Used in worst case analysis of algorithm performance.

$$f(n) = O(g(n))$$

iff there exist two positive constants c and n_0 such that

$$|f(n)| \leq c\,|g(n)| \quad \text{for all } n > n_0$$

$\Rightarrow$ i.e. g(n) is an upper bound on f(n)

$$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(n^3) < O(2^n) < O(n!)$$

i.e Notations for computational complexity of algorithms
+ rate of growth of functions

Asymptotic Notations for Algorithm Analysis

- **Asymptotic Lower bound:** Big Omega Notation $\boxed{\Omega(\,)}$

  Used in the analysis of the lower limit of algorithm performance

$$f(n) = \Omega(g(n))$$

if there exist positive constants c, n_0 such that

$$|f(n)| \geq c\,|g(n)| \qquad \text{for all} \quad n > n_0$$

$\Rightarrow$ i.e. g(n) is a lower bound on f(n)

- **Asymptotic Tight bound:** Big Theta Notation $\boxed{\Theta(\,)}$

  Used in finding a tight limit on algorithm performance

$$f(n) = \Theta(g(n))$$

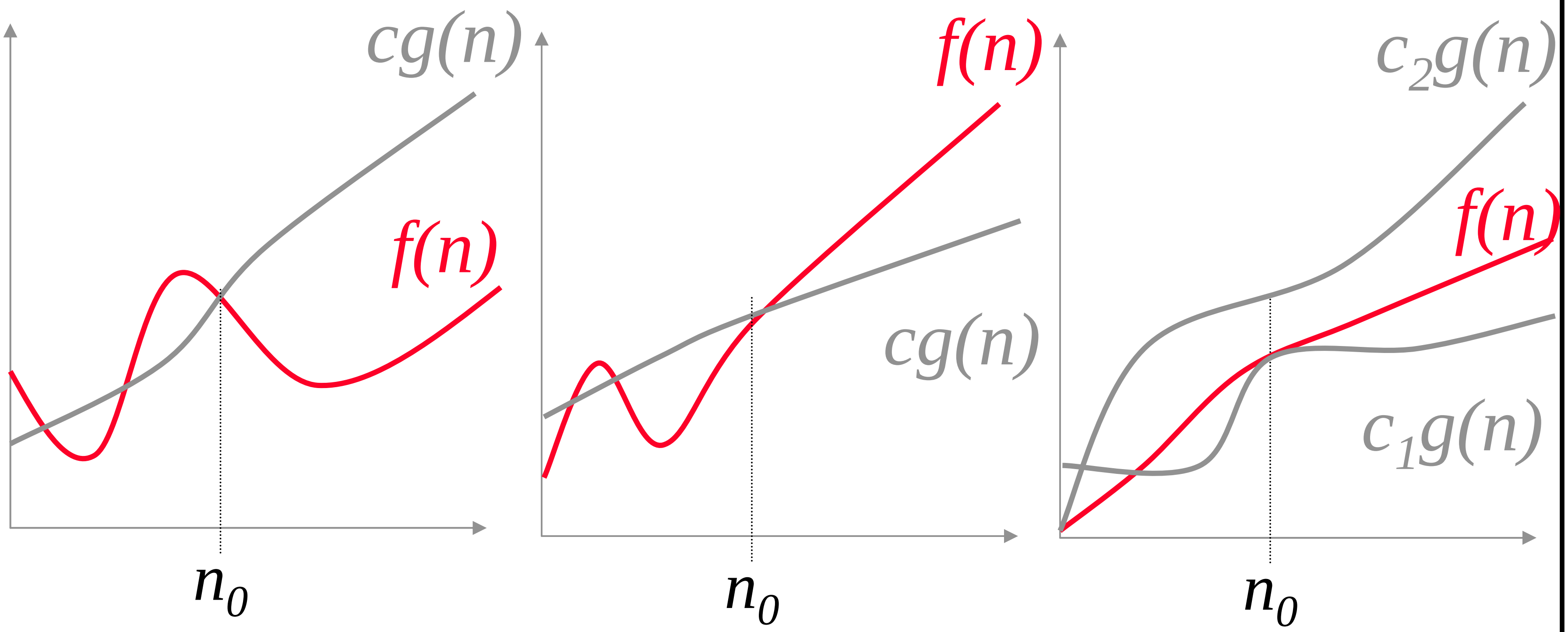
if there exist constant positive integers c_1, c_2, and n_0 such that

$$c_1\,|g(n)| \leq |f(n)| \leq c_2\,|g(n)| \qquad \text{for all} \quad n > n_0$$

$\Rightarrow$ i.e. g(n) is both an upper and lower bound on f(n)

AKA Tight bound

Graphs of O, Ω, Θ



f(n) =O(g(n))
Upper Bound

f(n) = Ω(g(n))
Lower Bound

f(n) = Θ(g(n))
Tight bound

Or other metric or quantity such as memory requirement, communication etc ..

Rate of Growth of Common Computing Time Functions

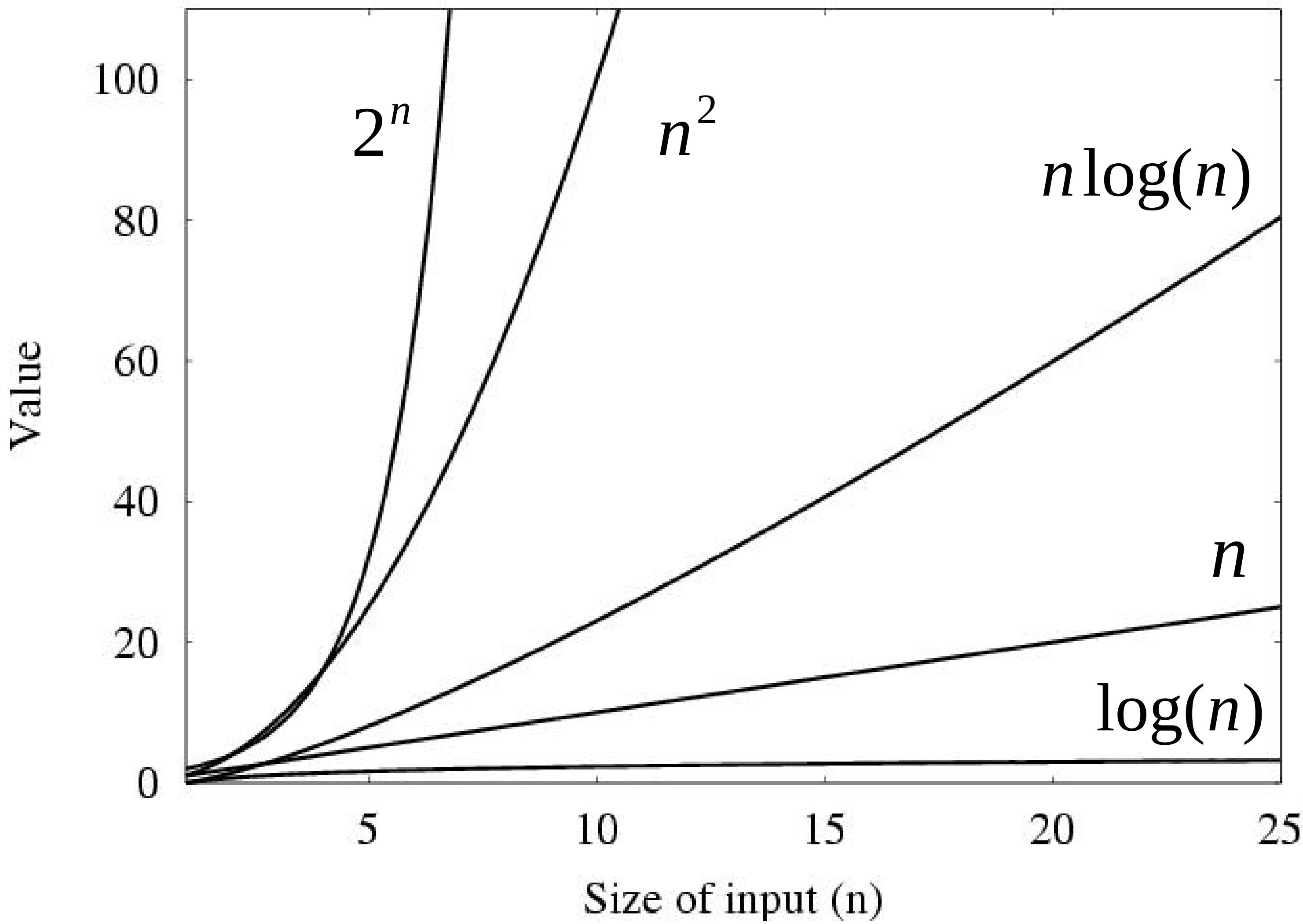
$\log_2 n$	n	$n \log_2 n$	n^2	n^3	2^n	$n!$
0	1	0	1	1	2	1
1	2	2	4	8	4	2
2	4	8	16	64	16	24
3	8	24	64	512	256	40320
4	16	64	256	4096	65536	20922789888000
5	32	160	1024	32768	4294967296	2.6×10^{35}

e.g NP-Complete/Hard Algorithms
(NP – Non Polynomial)

$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(n^3) < O(2^n) < O(n!)$

NP = Non-Polynomial

Rate of Growth of Common Computing Time Functions



O(1) < O(log n) < O(n) < O(n log n) < O (n^2) < O(n^3) < O(2^n)

PRAM: An Idealized Shared-Memory Parallel Computer Model

- Parallel Random-Access Machine (PRAM):
  - *p* processor, global shared memory model.
  - Models idealized parallel shared-memory computers with zero synchronization, communication or memory access overhead.
  - Utilized in parallel algorithm development and scalability and complexity analysis.

Why?

- PRAM variants: More realistic models than pure PRAM
  - EREW-PRAM: Simultaneous memory reads or writes to/from the same memory location are not allowed.
  - CREW-PRAM: Simultaneous memory writes to the same location is not allowed. (Better to model SAS MIMD?)
  - ERCW-PRAM: Simultaneous reads from the same memory location are not allowed.
  - CRCW-PRAM: Concurrent reads or writes to/from the same memory location are allowed.

Sometimes used to model SIMD since no memory is shared

Example: sum algorithm on P processor PRAM

Add n numbers

- **Input:** Array A of size $n = 2^k$ in shared memory
- **Initialized local variables:**
  - the order n,
  - number of processors $p = 2^q \leq n$,
  - the processor number s
- **Output:** The sum of the elements of A stored in shared memory

begin

1. for j = 1 to l (= n/p) do
   Set B(l(s - 1) + j): = A(l(s-1) + j)

p partial sums O(n/p)

2. for h = 1 to $\log_2 p$ do
   2.1 if (k- h - q ≥ 0) then
   for j = 2^{k-h-q}(s-1) + 1 to 2^{k-h-q}S do
   Set B(j): = B(2j -1) + B(2s)
   2.2 else {if (s ≤ 2^{k-h}) then
   Set B(s): = B(2s -1) + B(2s)}

Binary tree computation O ($\log_2$ p)

3. if (s = 1) then set S: = B(1)

end

Compute p partial Sums

Running time analysis:

- Step 1: takes O(n/p) each processor executes n/p operations
- The hth of step 2 takes O(n / (2^hp)) since each processor has to perform (n / (2^hp)) ¬ operations
- Step three takes O(1)
- Total running time:

$$T_p(n) = O\left(\frac{n}{p} + \sum_{h=1}^{\log p}\left\lceil \frac{n}{2^h p}\right\rceil\right) = O\left(\frac{n}{p} + \log p\right)$$

n >>p

Sequential Time $T_1(n) = O(n)$

Example: Sum Algorithm on P Processor PRAM

$T = 2 + \log_2(8) = 2 + 3 = 5$

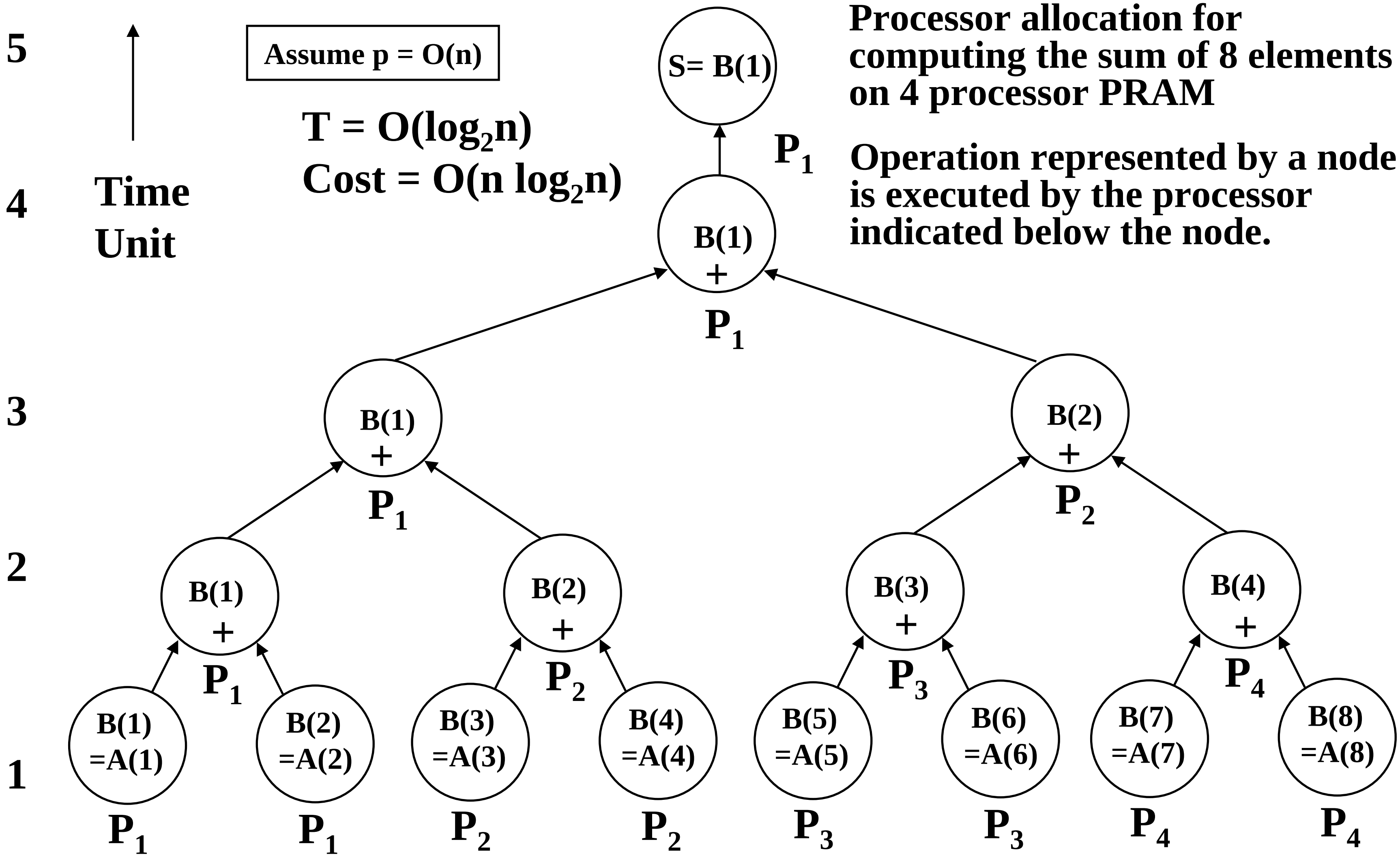
For $n = 8$ $p = 4 = n/2$

Processor allocation for computing the sum of 8 elements on 4 processor PRAM

Operation represented by a node is executed by the processor indicated below the node.

Assume p = O(n)

$T = O(\log_2 n)$
$\text{Cost} = O(n \log_2 n)$



For $n \gg p$ $O(n/p + \log_2 p)$

Performance of Parallel Algorithms

- Performance of a parallel algorithm is typically measured in terms of worst-case analysis. i.e using order notation O( )
- For problem Q with a PRAM algorithm that runs in time T(n) using P(n) processors, for an instance size of *n*:

Cost of a parallel algorithm

  – The time-processor product C(n) = T(n) . P(n) represents the cost of the parallel algorithm.
  – For $P \leq P(n)$, each of the of the T(n) parallel steps is simulated in O(P(n)/p) substeps. Total simulation takes O(T(n)P(n)/p) = O(C(n)/p)
  – The following four measures of performance are asymptotically equivalent:
    - P(n) processors and T(n) time
    - C(n) = P(n)T(n) cost and T(n) time
    - O(T(n)P(n)/p) time for any number of processors $p \leq P(n)$
    - O(C(n)/p + T(n)) time for any number of processors.

Illustrated next with an example

Matrix Multiplication On PRAM

- **Multiply matrices A x B = C of sizes n x n**
- **Sequential Matrix multiplication:**

For i=1 to n {

For j=1 to n { $C(i,j) = \sum_{t=1}^{n} a(i,t) \times b(t,j)$ **}}**

Dot product O(n) sequentially on one processor

PRAM Speedup: = $n^3 / \log_2 n$

Thus sequential matrix multiplication time complexity $O(n^3)$

- **Matrix multiplication on PRAM with p = $O(n^3)$ processors.**
  - **Compute in parallel for all i, j, t = 1 to n**

    c(i,j,t) = a(i, t) x b(t, j)   O(1)

    All product terms computed in parallel in one time step using n^3 processors
  - **Compute in parallel for all i;j = 1 to n:**

    $$C(i,j) = \sum_{t=1}^{n} c(i,j,t)$$ **$O(\log_2 n)$**

    All dot products computed in parallel Each taking $O(\log_2 n)$

  Thus time complexity of matrix multiplication on PRAM with n^3 processors = $O(\log_2 n)$ Cost(n) = $O(n^3 \log_2 n)$

  - **Time complexity of matrix multiplication on PRAM with n^2 processors = $O(n \log_2 n)$**
  - **Time complexity of matrix multiplication on PRAM with n processors = $O(n^2 \log_2 n)$**

From last slide: O(C(n)/p + T(n)) time complexity for any number of processors.

The Power of The PRAM Model

- Well-developed techniques and algorithms to handle many computational problems exist for the PRAM model.
- Removes algorithmic details regarding synchronization and communication cost, concentrating on the structural and *fundamental data dependency properties* (dependency graph) of the parallel computation/algorithm.
- Captures several important parameters of parallel computations. Operations performed in unit time, as well as processor allocation.
- The PRAM design paradigms are robust and many parallel network (message-passing) algorithms can be directly derived from PRAM algorithms.
- It is possible to incorporate synchronization and communication costs into the shared-memory PRAM model.

Network Model of Message-Passing Multicomputers

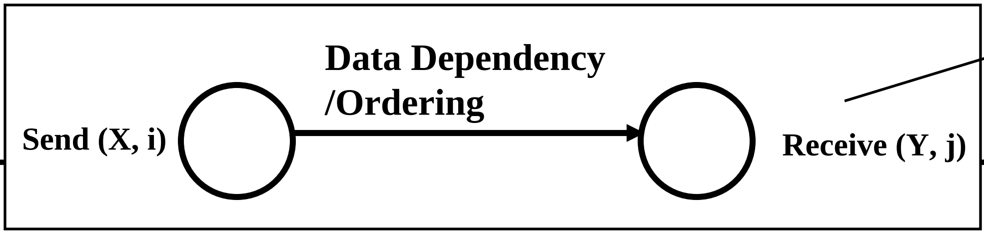
- A network of processors can viewed as a graph **G (N,E)**
  - Each node $i \in N$ represents a processor

    Graph represents network topology
  - Each edge $(i,j) \in E$ represents a two-way communication link between processors i and j.

    e.g Point-to-point interconnect
  - Each processor is assumed to have its own local memory.
  - No shared memory is available.
  - Operation is synchronous or asynchronous (using message passing).
  - <u>Basic message-passing communication primitives:</u>
    - **send(X,i)** a copy of data X is sent to processor P_i, execution continues.
    - **receive(Y, j)** execution of recipient processor is suspended (blocked or waits) until the data from processor P_j is received and stored in Y then execution resumes.

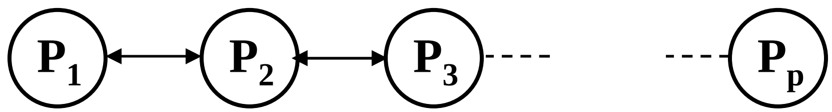
Blocking Receive

i.e Blocking Receive



Network Model of Multicomputers

- Routing is concerned with delivering each message from source to destination over the network. Path or route message takes in network
- Additional important network topology parameters:
  - The **network diameter** is the maximum distance between any pair of nodes (in links or hops). i.e length of longest route between any two nodes
  - The **maximum degree** of any node in G
    - Directly connected to how many other nodes
- Example:



  - Linear array: P processors $P_1, \ldots, P_p$ are connected in a linear array where:
    - Processor P_i is connected to P_{i-1} and P_{i+1} if they exist.
    - Diameter is p-1; maximum degree is 2 (1 or 2).
  - A ring is a linear array of processors where processors P_1 and P_p are directly connected. Degree = 2, Diameter = p/2

Example Network Topology:

A Four-Dimensional (4D) Hypercube

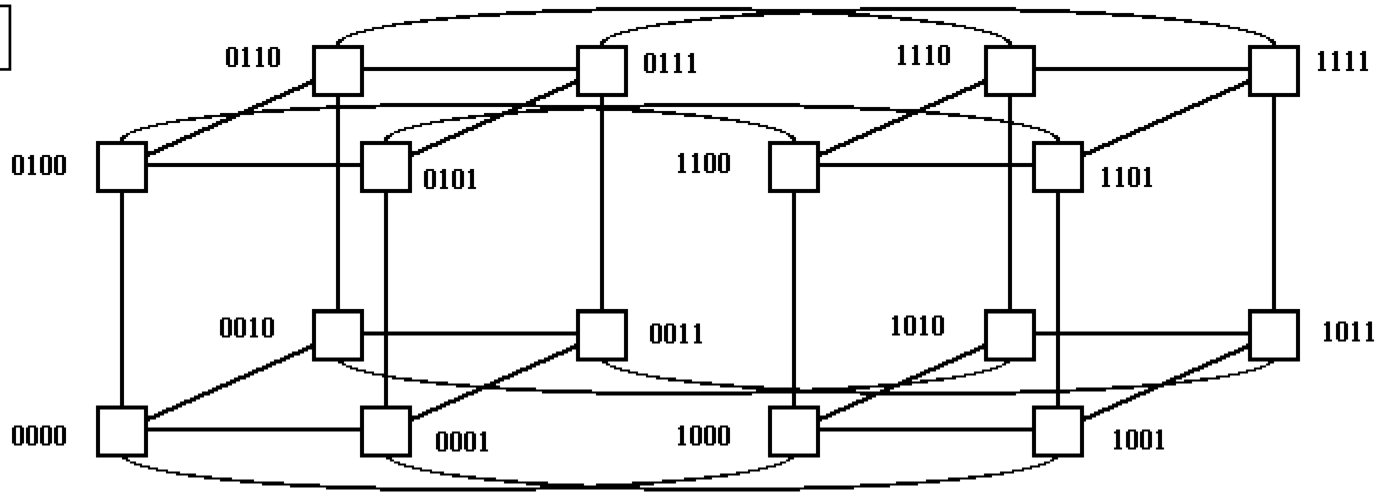
- In a d-dimensional binary hypercube, each of the $N = 2^d$ nodes is assigned a d-bit address.

Connectivity

- Two processors are connected if their binary addresses differ in one bit position.
- Degree = Diameter = $d = \log_2 N$

here d = 4

Example d=4



Here: d = Degree = Diameter = 4
Number of nodes = $N = 2^d = 2^4$ = 16 nodes

Binary tree computations map directly to the hypercube topology

Dimension order routing example node 0000 to 1111:
$0000 \xrightarrow{1} 0001 \xrightarrow{2} 0011 \xrightarrow{3} 0111 \xrightarrow{4} 1111$ (4 hops)

Example: Message-Passing Matrix Vector Product on a Ring

Sequentially = $O(n^2)$

$Y = Ax$

- Input:
  - n x n matrix A ; vector x of order n
  - The processor number i. The number of processors
  - The ith submatrix $B = A(1:n, (i-1)r+1 ; ir)$ of size n x r where $r = n/p$
  - The ith subvector $w = x(i-1)r + 1 : ir)$ of size r
- Output:
  - Processor P_i computes the vector $y = A_1x_1 + \ldots. A_ix_i$ and passes the result to the right
  - Upon completion P_1 will hold the product Ax

n >> p
n multiple of p

Begin

T_{comp}

1. Compute the matrix vector product z = Bw

For all processors, $O(n^2/p)$

T_{comm}

2. If i = 1 then set y: = 0
   else receive(y,left)
3. Set y: = y +z
4. send(y, right)
5. if i =1 then receive(y,left)

O(np)

End

$T_{comp} = k(n^2/p)$
$T_{comm} = p(l+ mn)$
$T = T_{comp} + T_{comm}$
$= k(n^2/p) + p(l+ mn)$

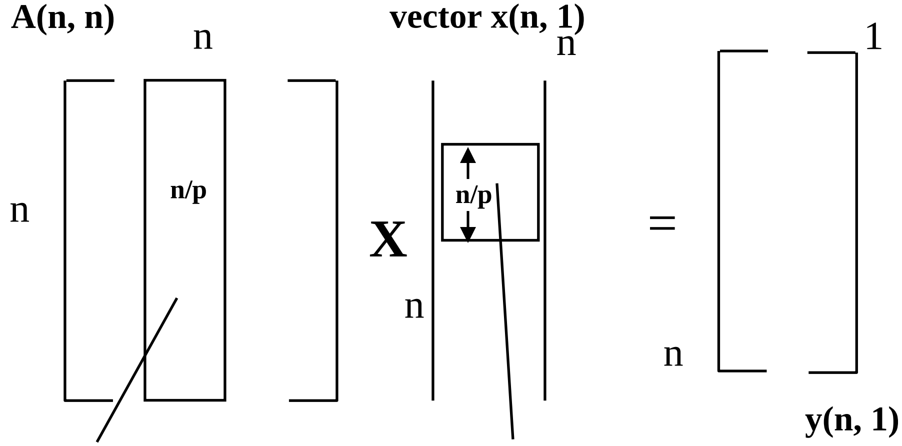
Comp = Computation

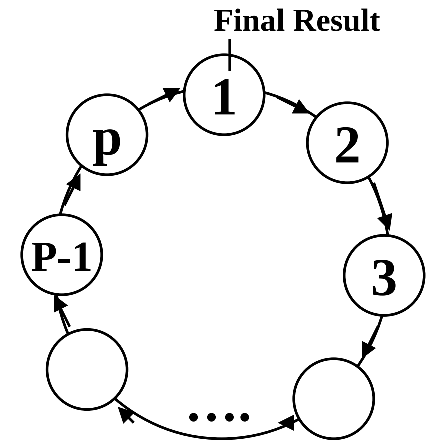
Comm = Communication

k, l, m constants

Sequential time complexity of Y = Ax : $O(n^2)$

Matrix Vector Product y = Ax on a Ring





Submatrix B_i of size n x n/p **Subvector W_i of size (n/p,1)**

p processors

(n,1) $Z_i = B_i w_i$ **Compute in parallel**

$$Y = Ax = Z_1 + Z_2 + \dots Zp = B_1 w_1 + B_2 w_2 + \dots B_p w_p$$

Communication-to-Computation Ratio = $= pn / (n^2/p) = p^2/n$

Computation: $O(n^2/p)$

- **For each processor i compute in parallel**

  $Z_i = B_i w_i$ $O(n^2/p)$

Communication $O(pn)$

- **For processor #1: set y =0, y = y + Zi, send y to right processor**
- **For every processor except #1**

  Receive y from left processor, y = y + Zi, send y to right processor
- **For processor 1 receive final result from left processor p**
- $$T = O(n^2/p + pn)$$

Computation **Communication**

Creating a Parallel Program

- **Assumption: Sequential algorithm to solve problem is given**
  - **Or a different algorithm with <u>more inherent parallelism is devised</u>.**
  - **Most programming problems have several parallel solutions or algorithms. The best solution may differ from that suggested by existing sequential algorithms.**

One must: **Computational Problem → Parallel Algorithm → Parallel Program**

- **Identify work that can be done in parallel (dependency analysis)**
- **Partition work and perhaps data among processes (Tasks)** — **Determines size and number of tasks**
- **Manage data access, communication and synchronization**
- ***Note*: work includes computation, data access and I/O**

Main goal: Maximize Speedup **of parallel processing**

By:
<u>1- Minimizing parallelization overheads</u>
<u>2- Balancing workload on processors</u>

$$\textbf{Speedup } (p) = \frac{Performance(p)}{Performance(1)}$$

For a fixed size problem:

$$\textbf{Speedup } (p) = \frac{Time(1)}{Time(p)}$$

The processor with max. execution time determines parallel execution time

***Time (p)* = *Max* (Work + Synch Wait Time + Comm Cost + Extra Work)**

Hardware Vs. Software Parallelism

Hardware DOP

- **Hardware parallelism:** e.g Number of processors

  i.e hardware-supported threads of execution

  At Thread Level Parallelism (TLP)

  – Defined by machine architecture, hardware multiplicity (number of processors available) and connectivity.
  – Often a function of cost/performance tradeoffs.
  – Characterized in a single processor by the number of instructions *k* issued in a single cycle (*k*-issue processor).
  – A multiprocessor system with *n* *k*-issue processor can handle a maximum limit of *nk* parallel instructions (at ILP level) or n parallel threads at thread-level parallelism (TLP) level.

Software DOP

- **Software parallelism:** e.g Degree of Software Parallelism (DOP) or number of parallel tasks at selected task or grain size at a given time in the parallel computation
  – Defined by the control and data dependence of programs.
  – Revealed in program profiling or program dependency (data flow) graph. → i.e. number of parallel tasks at a given time
  – A function of algorithm, parallel task grain size, programming style and compiler optimization.

DOP = Degree of Parallelism

Levels of Software Parallelism in Program Execution

According to task (grain) size

Increasing communications demand and mapping/scheduling overheads

Higher C-to-C Ratio

Level 5	Jobs or programs (Multiprogramming) — i.e multi-tasking	Coarse Grain
Level 4	Subprograms, job steps or related parts of a program	Coarse Grain / Medium Grain
Level 3	Procedures, subroutines, or co-routines	Medium Grain
Level 2	Non-recursive loops or unfolded iterations	Fine Grain
Level 1	Instructions or statements	Fine Grain

Thread Level Parallelism (TLP)

Instruction Level Parallelism (ILP)

Higher Degree of Software Parallelism (DOP)

More Smaller Tasks

Task size affects Communication-to-Computation ratio (C-to-C ratio) and communication overheads

Computational Parallelism and Grain Size

- **Task grain size (granularity) is a measure of the amount of computation involved in a task in parallel computation:**
  - **Instruction Level (Fine Grain Parallelism):**
    - **At instruction or statement level.**
    - **20 instructions grain size or less.**
    - **For scientific applications, parallelism at this level range from 500 to 3000 concurrent statements**
    - **Manual parallelism detection is difficult but assisted by parallelizing compilers.**
  - **Loop level (Fine Grain Parallelism):**
    - **Iterative loop operations.**
    - **Typically, 500 instructions or less per iteration.**
    - **Optimized on vector parallel computers.**
    - **Independent successive loop operations can be vectorized or run in SIMD mode.**

Computational Parallelism and Grain Size

- **Procedure level (Medium Grain Parallelism): :**
  - Procedure, subroutine levels.
  - Less than 2000 instructions.
  - More difficult detection of parallel than finer-grain levels.
  - Less communication requirements than fine-grain parallelism.
  - Relies heavily on effective operating system support.
- **Subprogram level (Coarse Grain Parallelism): :**
  - Job and subprogram level.
  - Thousands of instructions per grain.
  - Often scheduled on message-passing multicomputers.
- **Job (program) level, or Multiprogrammimg:**
  - Independent programs executed on a parallel computer.
  - Grain size in tens of thousands of instructions.

Software Parallelism Types: Data Vs. Functional Parallelism

1 - Data Parallelism:

– Parallel (often similar) computations performed on elements of large data structures
  - (e.g numeric solution of linear systems, pixel-level image processing)

– Such as resulting from parallelization of loops.

– Usually easy to load balance.

i.e max DOP

– Degree of concurrency usually increases with input or problem size. e.g $O(n^2)$ in equation solver example (next slide).

2- Functional Parallelism:

- Entire large tasks (procedures) with possibly different functionality that can be done in parallel on the same or different data.
  - – Software Pipelining: Different functions or software stages of the pipeline performed on different data:
    - As in video encoding/decoding, or polygon rendering.
- Concurrency degree usually modest and does not grow with input size (i.e. problem size)
  - – Difficult to load balance.
  - – Often used to reduce synch wait time between data parallel phases.

Most scalable parallel computations/programs:

(more concurrency as problem size increases) parallel programs:

Data parallel computations/programs (per this loose definition)

– Functional parallelism can still be exploited to reduce synchronization wait time between data parallel phases.

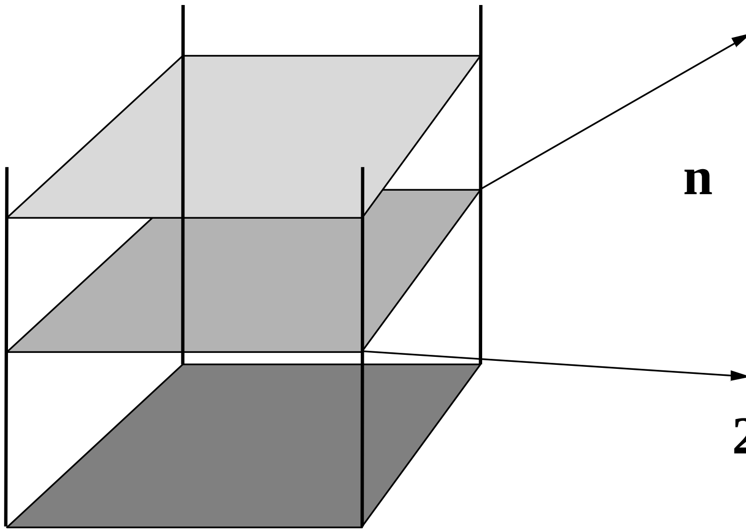
Actually covered in PCA 3.1.1 page 124

Concurrency = Parallelism

Example Motivating Problem: Simulating Ocean Currents/Heat Transfer ...

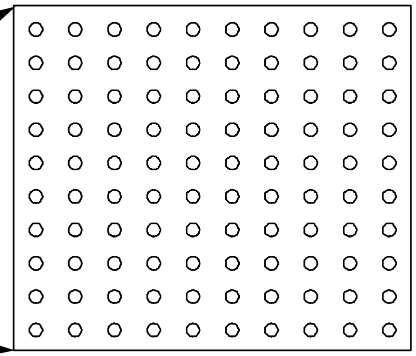
With High Degree of Data Parallelism

n grids



(a) Cross sections

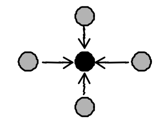
n



n

2D Grid
n x n

(b) Spatial discretization of a cross section



Expression for updating each interior point:

$$A[i,j] = 0.2 \times (A[i,j] + A[i,j-1] + A[i-1,j] + A[i,j+1] + A[i+1,j])$$

Total $O(n^3)$ Computations Per iteration

<u>Maximum Degree of Parallelism (DOP) or concurrency:</u> $O(n^2)$ <u>data parallel</u> computations per 2D grid per iteration

When one task updates/computes one grid element

- **Model as two-dimensional nxn grids**
- **Discretize in space and time**
  - **finer spatial and temporal resolution => greater accuracy**
- **Many different computations per time step** — **$O(n^2)$ per grid**
  - **set up and solve equations iteratively (Gauss-Seidel) .**
- **Concurrency across and within grid computations per iteration**
  - **n^2 parallel computations per grid x number of grids**

Covered next lecture (PCA 2.3)

Limited Concurrency: Amdahl's Law

- Most fundamental limitation on parallel speedup.
- Assume $\boxed{\text{a fraction s}}$ of sequential execution time runs on a single processor and $\boxed{\text{cannot be parallelized.}}$

DOP=1

i.e sequential or serial portion of the computation

- Assuming that the problem size remains fixed and that the remaining $\boxed{\text{fraction (1-s)}}$ can be parallelized without any parallelization overheads to $\boxed{\text{run on p processors}}$ and thus reduced by a factor of p.
- The resulting speedup for p processors:

DOP=p

i.e. perfect speedup for parallelizable portion

i.e. two degrees of parallelism: 1 or p

$$\text{Speedup(p)} = \frac{\text{Sequential Execution Time}}{\text{Parallel Execution Time}} \qquad \frac{T(1)}{T(p)}$$

Parallel Execution Time = (S + (1-S)/P) X Sequential Execution Time

T(p) T(1)

$$\text{Speedup(p)} = \frac{\cancel{\text{Sequential Execution Time}}}{(\text{s} + (1\text{-s})/\text{p}) \text{ X } \cancel{\text{Sequential Execution Time}}} = \frac{1}{\text{s} + (1\text{-s})/\text{p}}$$

+ Parallelization Overheads ?

- Thus for a fixed problem size, if fraction *s* of sequential execution is inherently serial, speedup $\leq 1/s$

Fixed problem size speedup

Limit on Speedup

Amdahl's Law Example

- **<u>Example:</u> 2-Phase n-by-n Grid Computation** e.g data parallel DOP = $O(n^2)$

Phase 1 – Sweep over *n*-by-*n* grid and do some independent computation

Phase 2 – Sweep again and add each value to <u>global sum</u> sequentially

Parallel 1 – Time for first phase = n^2/p *P* = number of processors

Sequential 2 – Second phase serialized at global variable, so time = n^2

Sequential time = Time Phase1 + Time Phase2 = $n^2 + n^2 = 2n^2$

Here s = 0.5

$$\frac{T(1)}{T(p)} = \text{Speedup} \le \frac{2n^2}{\underbrace{\frac{n^2}{p}}_{\text{Phase1 (parallel)}} + \underbrace{n^2}_{\text{Phase2 (sequential)}}} = \frac{1}{\frac{0.5}{p} + 0.5}$$

or at most $1/s = 1/.5 = 2$ Max. speedup

– **Possible Trick:** divide second "sequential" phase into two:

1 • Accumulate into <u>private sum</u> during sweep Time = n^2/p

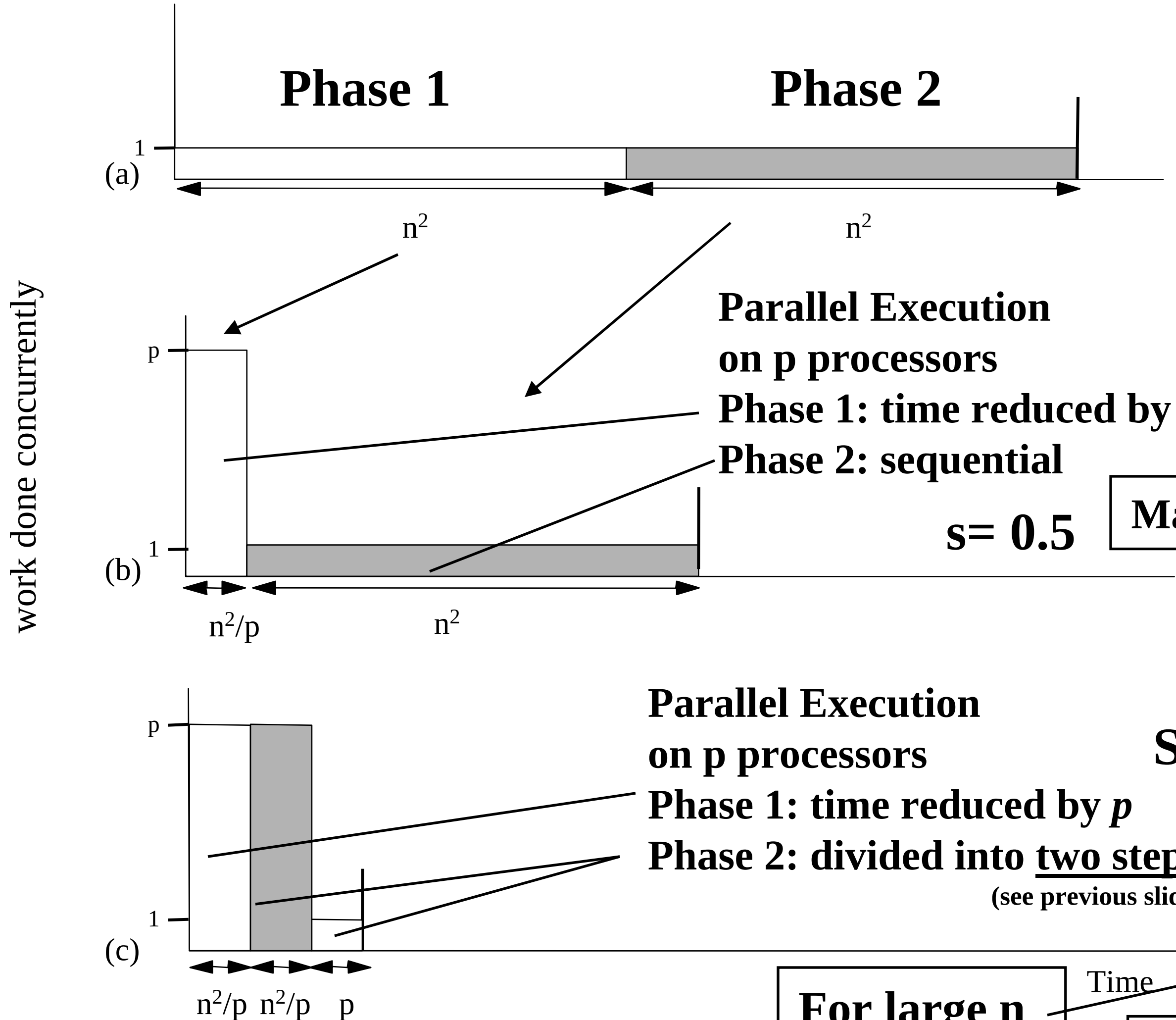
2 • Add per-process private sum into global sum Time = p

– Total parallel time is $n^2/p + n^2/p + p$, and speedup:

$$\text{Speedup} = \frac{2n^2}{2n^2/p + p} = \frac{1}{1/p + p/2n^2}$$

For large n Speedup ~ *p*

Amdahl's Law Example:
2-Phase n-by-n Grid Computation
A Pictorial Depiction



Sequential Execution

Parallel Execution on p processors
Phase 1: time reduced by *p*
Phase 2: sequential

s= 0.5

$$\textbf{Speedup=} \frac{1}{\frac{0.5}{p} + 0.5}$$

Maximum Possible Speedup = 2

Parallel Execution on p processors
Phase 1: time reduced by *p*
Phase 2: divided into <u>two steps</u>
(see previous slide)

$$\textbf{Speedup=} \frac{2n^2}{2n^2 + p^2} = \frac{1}{1/p + p/2n^2}$$

For large n
Speedup ~ *p*

Parallel Performance Metrics
Degree of Parallelism (DOP)

- **For a given time period, DOP reflects the number of processors in a specific parallel computer actually executing a particular parallel program.**

i.e DOP at a given time = Min (Software Parallelism, Hardware Parallelism)

- **<u>Average Degree of Parallelism A:</u>**
  - **given maximum parallelism = *m***
  - ***n* homogeneous processors**
  - **computing capacity of a single processor Δ** ← **Computations/sec**
  - **Total amount of work W (instructions, computations):**

$$W = \Delta \int_{t1}^{t2} DOP(t)\,dt$$ **or as a discrete summation** $$W = \Delta \sum_{i=1}^{m} i.t_i$$

Where t_i is the total time that DOP = i and $\sum_{i=1}^{m} t_i = t_2 - t_1$

The average parallelism A:

$$A = \frac{1}{t_2 - t_1} \int_{t1}^{t2} DOP(t)\,dt$$

In discrete form

$$A = \left(\sum_{i=1}^{m} i.t_i \right) \Big/ \left(\sum_{i=1}^{m} t_i \right)$$

DOP Area

Parallel Execution Time

i.e A = Total Work / Total Time = W / T

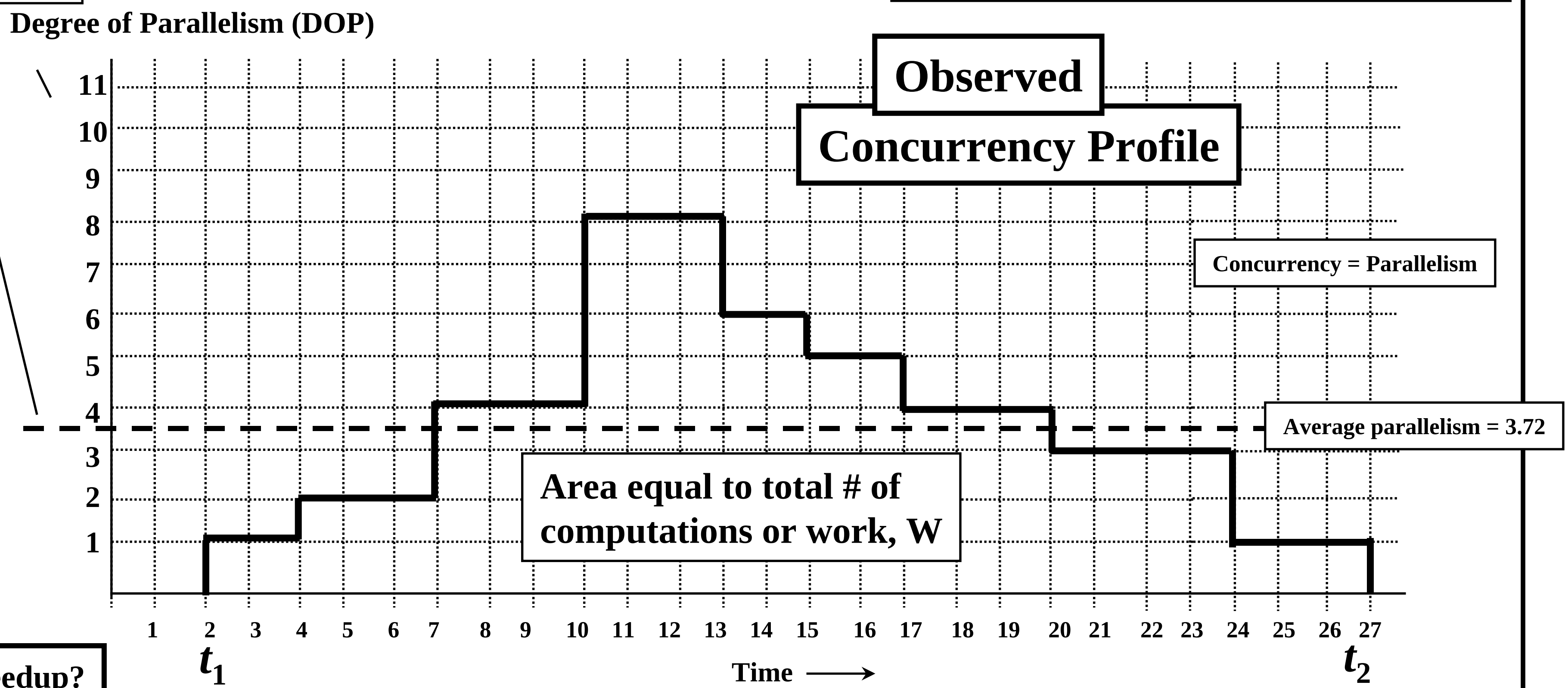
Example: Concurrency Profile of A Divide-and-Conquer Algorithm

- Execution observed from $t_1 = 2$ to $t_2 = 27$
- Peak parallelism (i.e. peak DOP) $m = 8$
- A = (1x5 + 2x3 + 3x4 + 4x6 + 5x2 + 6x2 + 8x3) / (5 + 3+4+6+2+2+3)
  = 93/25 = 3.72

$$A = \left(\sum_{i=1}^{m} i \cdot t_i\right) \Bigg/ \left(\sum_{i=1}^{m} t_i\right) = \frac{\text{Work}}{\text{Time}}$$

Average Parallelism

A = Total Work / Total Time = W / T



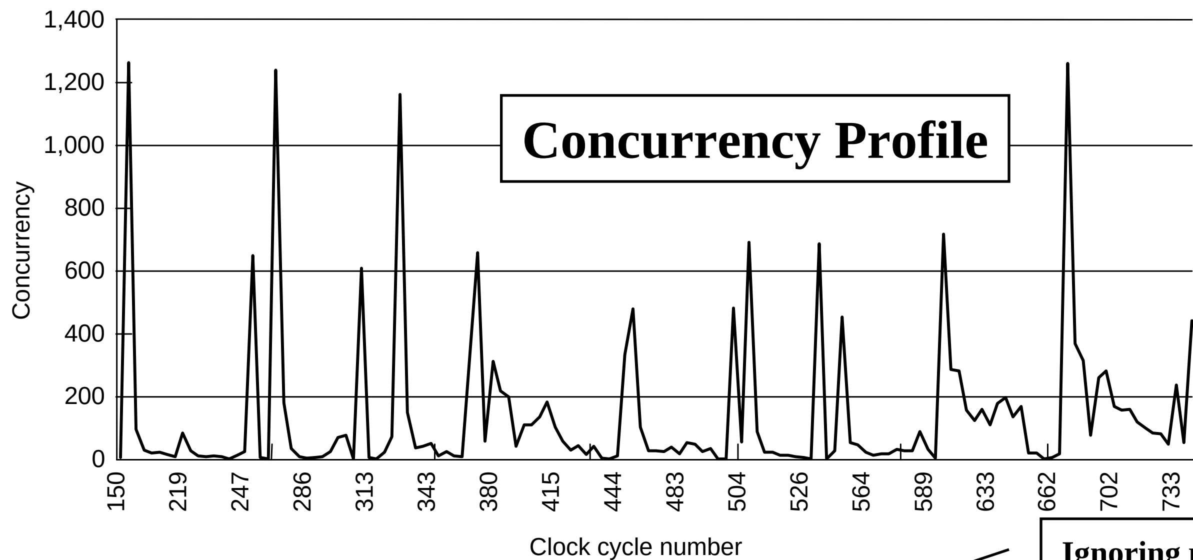
Speedup?

Ignoring parallelization overheads, what is the speedup?

(i.e total work on parallel processors equal to work on single processor)

Concurrency Profile & Speedup

For a parallel program DOP may range from 1 (serial) to a maximum *m*



Ignoring parallelization overheads

- **Area under curve is total work done, or time with 1 processor**
- **Horizontal extent is lower bound on time (infinite processors)**
- **Speedup is the ratio:** $\dfrac{\sum_{k=1}^{\infty} f_k k}{\sum_{k=1}^{\infty} f_k \left\lceil \frac{k}{p} \right\rceil}$**, base case:** $\dfrac{1}{s + \frac{1-s}{p}}$

  (for p processors)

  ***k* = degree of parallelism** **f_k = Total time with degree of parallelism k**

- **Amdahl's law applies to any overhead, not just limited concurrency.**

Ignoring parallelization overheads

Amdahl's Law with Multiple Degrees of Parallelism

- **Assume different fractions of sequential execution time of a problem running on a single processor have different degrees of parallelism (DOPs) and that the problem size remains fixed.**
  - **Fraction F_i of the sequential execution time can be parallelized without any parallelization overheads to run on S_i processors and thus reduced by a factor of S_i.**
  - **The remaining fraction of sequential execution time cannot be parallelized and runs on a single processor.**
- **Then**

i.e. Sequential Execution Time

$$\frac{T(1)}{T(p)} = \textit{Speedup} = \frac{\cancel{\text{Original Execution Time}}\ \boxed{T(1)}}{\left(\left(1-\sum_i F_i\right)+\sum_i \frac{F_i}{S_i}\right) X\ \cancel{\text{Original Execution Time}}\ \boxed{T(1)}}$$

Fixed problem size speedup

Sequential Fraction (DOP=1)

$$\textit{Speedup} = \frac{1}{\left(\left(1-\sum_i F_i\right)+\sum_i \frac{F_i}{S_i}\right)}$$

Sequential Fraction (DOP=1)

+ overheads?

Note: All fractions F_i refer to original sequential execution time on one processor.

How to account for parallelization overheads in above speedup?

Amdahl's Law with Multiple Degrees of Parallelism : Example

- **Given the following degrees of parallelism in a program or computation**

i.e on 10 processors

$DOP_1 = S_1 = 10$ Percentage$_1$ = F_1 = 20%
$DOP_2 = S_2 = 15$ Percentage$_1$ = F_2 = 15%
$DOP_3 = S_3 = 30$ Percentage$_1$ = F_3 = 10%
$DOP_4 = S_4 = 1$ Percentage$_1$ = F_4 = 55%

Sequential Portion

- **What is the parallel speedup when running on a parallel system without any parallelization overheads ?**

$$Speedup = \frac{1}{\left((1 - \sum_i F_i) + \sum_i \frac{F_i}{S_i}\right)}$$

Sequential Portion

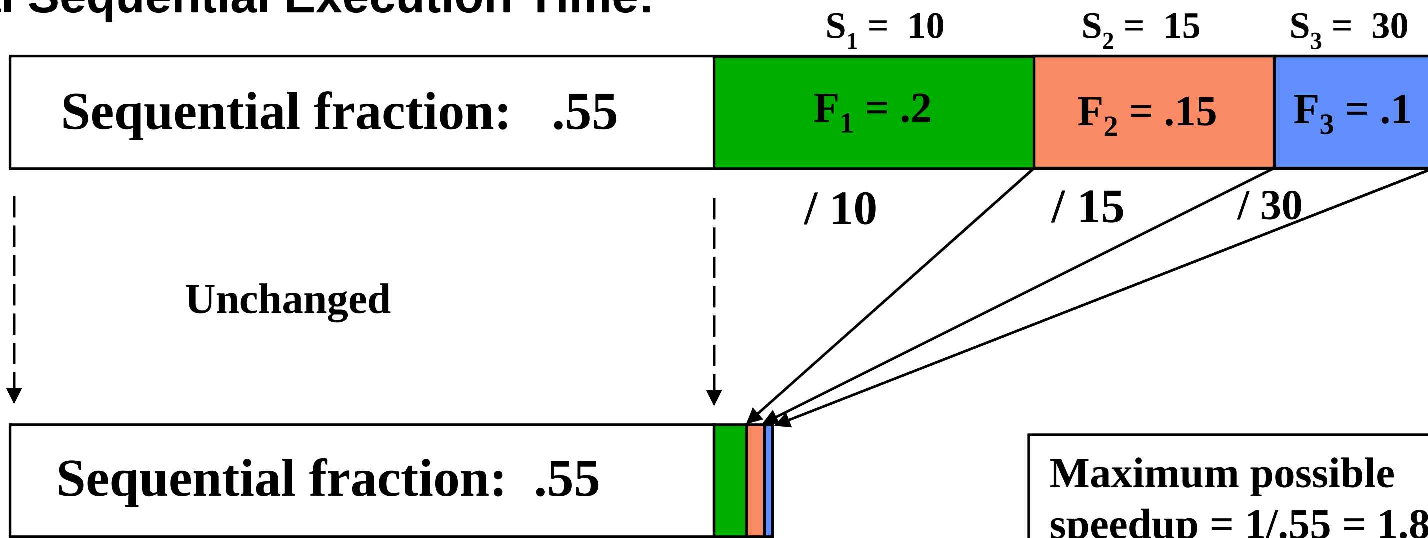
- **Speedup = 1 / [(1 - .2 - .15 - .1) + .2/10 + .15/15 + .1/30)]**
= 1 / [.55 + .0333]
= 1 / .5833 = 1.71

Maximum Speedup = 1/0.55 = 1.81 (limited by sequential portion)

From 551

Pictorial Depiction of Example

Before:
Original Sequential Execution Time:



Maximum possible speedup = 1/.55 = 1.81

Limited by sequential portion

After:
Parallel Execution Time: .55 + .02 + .01 + .00333 = .5833

Parallel Speedup = 1 / .5833 = 1.71

Note: All fractions (F_i, i = 1, 2, 3) refer to original sequential execution time.

What about parallelization overheads?

From 551

Parallel Performance Example

- The execution time T for three parallel programs is given in terms of processor count P and problem size N (or three parallel algorithms for a problem)
- In each case, we assume that the total computation work performed by an optimal sequential algorithm scales as $N+N^2$.

1 **For first parallel algorithm:** $T = N + N^2/P$

This algorithm partitions the computationally demanding $O(N^2)$ component of the algorithm but replicates the $O(N)$ component on every processor. There are no other sources of overhead.

2 **For the second parallel algorithm:** $T = (N+N^2)/P + 100$

This algorithm optimally divides all the computation among all processors but introduces an additional cost of 100.

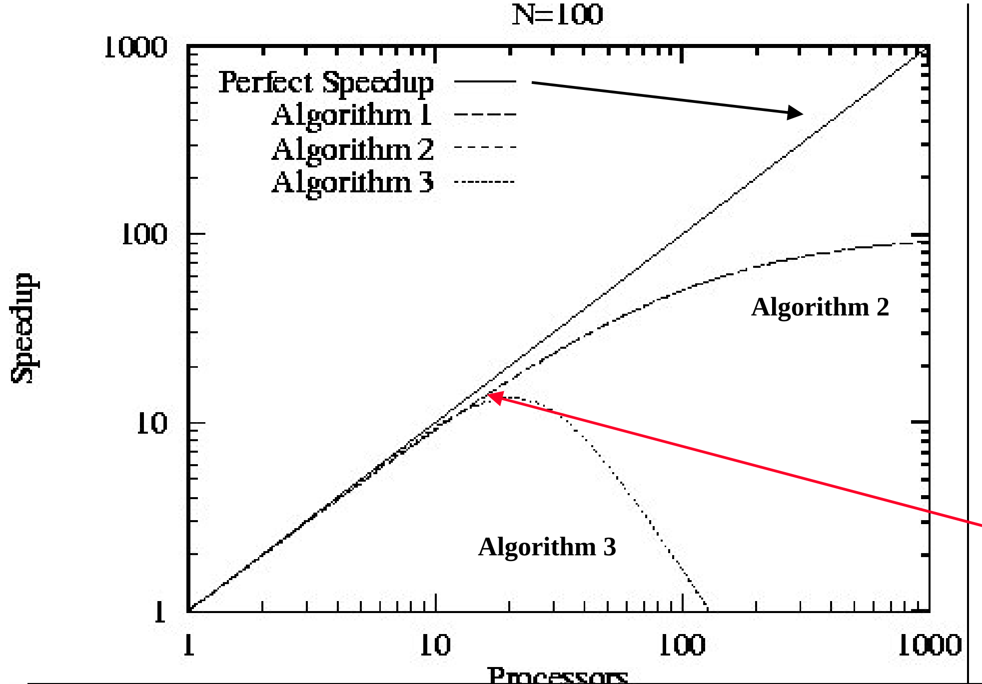
3 **For the third parallel algorithm:** $T = (N+N^2)/P + 0.6P^2$

This algorithm also partitions all the computation optimally but introduces an additional cost of $0.6P^2$.

- All three algorithms achieve a speedup of about 10.8 when P = 12 and N=100 . However, they behave differently in other situations as shown next.
- With N=100 , all three algorithms perform poorly for larger P , although Algorithm (3) does noticeably worse than the other two.
- When N=1000 , Algorithm (2) is much better than Algorithm (1) for larger P .

N = Problem Size P = Number of Processors

Parallel Performance Example (continued)



All algorithms achieve:
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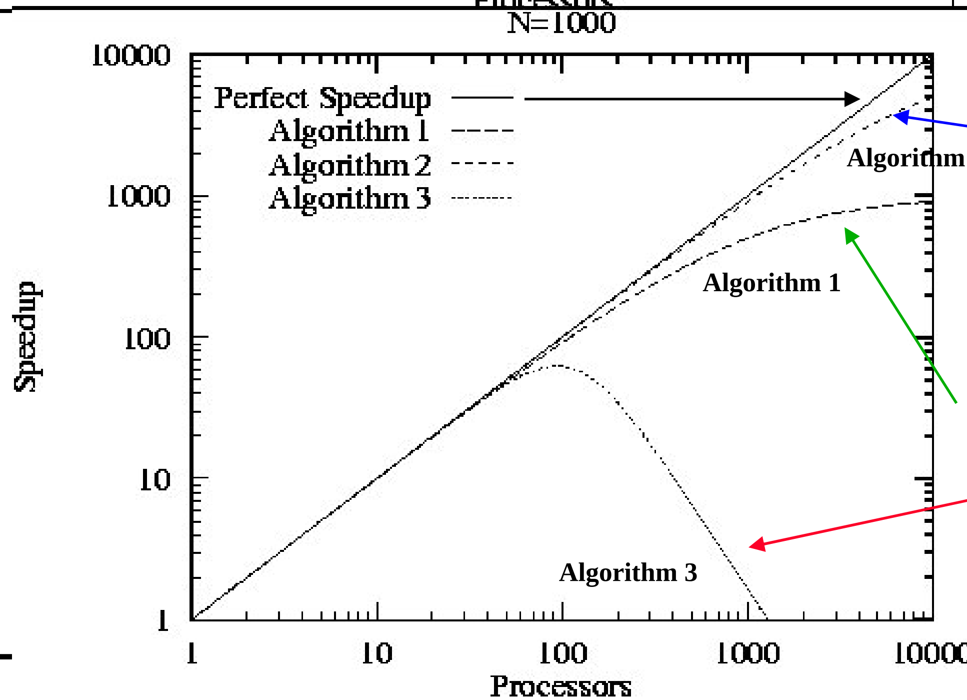

N= Problem Size
P = Number of processors

N=1000 , Algorithm (2) performs much better than Algorithm (1) for larger P .

Algorithm 1: T = $N + N^2/P$
Algorithm 2: T = $(N+N^2)/P + 100$
Algorithm 3: T = $(N+N^2)/P + 0.6P^2$

Slide 31 Repeated

Creating a Parallel Program

- **Assumption: Sequential algorithm to solve problem is given**
  - **Or a different algorithm with <u>more inherent parallelism is devised</u>.**
  - **Most programming problems have several parallel solutions or algorithms. The best solution may differ from that suggested by existing sequential algorithms.**

One must: **Computational Problem → Parallel Algorithm → Parallel Program**

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Main goal: Maximize Speedup of parallel processing

By:
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2- Balancing workload on processors

$$\textit{Speedup}\ (p) = \frac{Performance(p)}{Performance(1)}$$

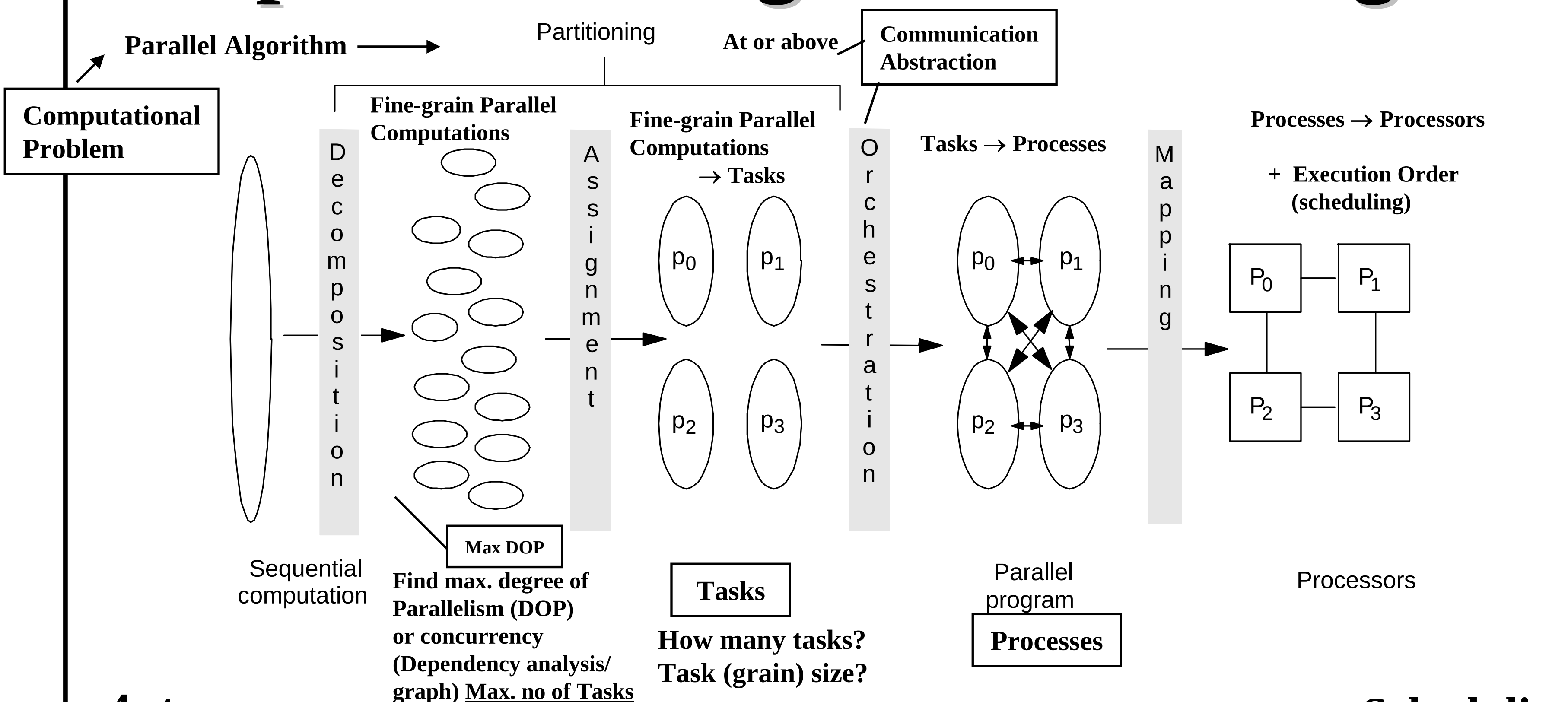
For a fixed size problem:

$$\textit{Speedup}\ (p) = \frac{Time(1)}{Time(p)}$$

The processor with max. execution time determines parallel execution time

Time (p) = *Max* (Work + Synch Wait Time + Comm Cost + Extra Work)

Steps in Creating a Parallel Program



4 steps: **+ Scheduling**

1- Decomposition, 2- Assignment, 3- Orchestration, 4- Mapping

- **Done by programmer or system software (compiler, runtime, ...)**
- **Issues are the same, so assume programmer does it all explicitly** Vs. implicitly by parallelizing compiler

Computational Problem → Parallel Algorithm → Parallel Program

Partitioning: Decomposition & Assignment

Decomposition *Dependency Analysis/graph*

- **Break up computation into maximum number of small concurrent** i.e. parallel **computations that can be combined into fewer/larger tasks in assignment step:**
  - **Tasks may become available dynamically.**
  - **No. of available tasks may vary with time.**
  - **Together with assignment, also called *partitioning.***

Grain (task) size Problem (Assignment)

i.e. identify concurrency (dependency analysis) and decide level at which to exploit it.

i.e Find maximum software concurrency or parallelism (Decomposition)

(Task) Assignment

- **Grain-size problem:** Task Size? How many tasks?
  - **To determine the number and size of grains or tasks in a parallel program.**
  - **Problem and machine-dependent.**
  - **Solutions involve tradeoffs between parallelism, communication and scheduling/synchronization overheads.**
- **Grain packing:**
  - **To combine multiple fine-grain nodes (parallel computations) into a coarse grain** i.e larger **node (task) to reduce communication delays and overall scheduling overhead.**

Goal: Enough tasks to keep processors busy, but not too many (too much overheads)

- **No. of tasks available at a time is upper bound on achievable speedup**

+ Good load balance in mapping phase

Task

Assignment

Fine-Grain Parallel Computations → Tasks

To Maximize Speedup

- Specifying mechanisms to divide work up among tasks/processes:
  - Together with decomposition, also called ***partitioning***. *.. of dependency graph*
  - Balance workload, reduce communication and management cost
    - May involve duplicating computation to reduce communication cost.
- Partitioning problem:

Partitioning = Decomposition + Assignment

  - To partition a program into parallel tasks to give the shortest possible execution time on a specific parallel architecture.
    - Determine size and number of tasks in parallel program
- Structured approaches usually work well:
  - Code inspection (parallel loops) or understanding of application.
  - Well-known heuristics.
  - *Static* versus *dynamic* assignment.
- As programmers, we worry about partitioning first:
  - *Usually* independent of architecture or programming model.
  - But cost and complexity of using primitives may affect decisions.

Number of processors?

Communication/Synchronization Primitives used in orchestration

Orchestration

Tasks → Processes

Or threads

Done at or above Communication Abstraction

- **For a given parallel programming environment that realizes a parallel programming model, orchestration includes:**
  - **Naming data.**
  - **Structuring communication (using communication primitives)**
  - **Synchronization (ordering using synchronization primitives).**
  - **Organizing data structures and scheduling tasks temporally.**

Execution order (schedule)

- **Goals**

overheads

  - **Reduce cost of communication and synchronization as seen by processors**
  - **Preserve locality of data reference (includes data structure organization)**
  - **Schedule tasks to satisfy dependencies as early as possible**
  - **Reduce overhead of parallelism management.**

Closer?

- **Closest to architecture (and programming model & language).**
  - **Choices depend a lot on communication abstraction, efficiency of primitives.**
  - **Architects should provide appropriate primitives efficiently.**

Mapping/Scheduling

Processes → Processors
+ Execution Order (scheduling)

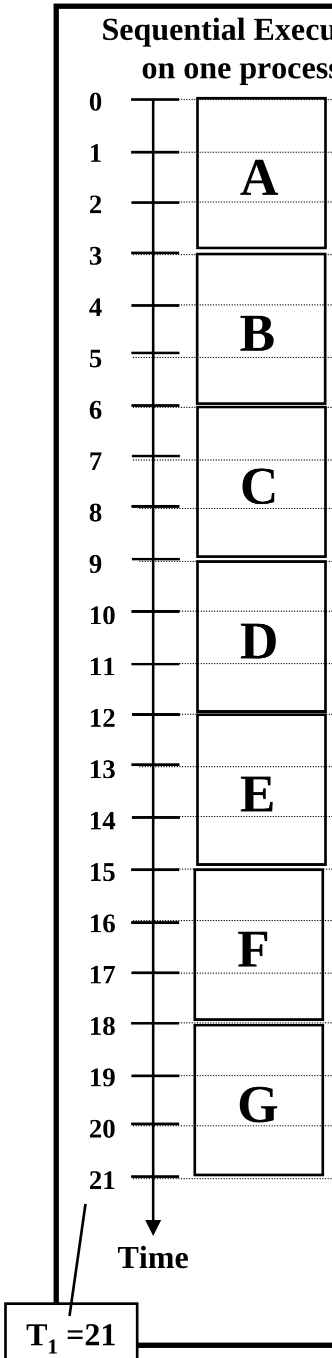
- **Each task is assigned to a processor in a manner that attempts to satisfy the competing goals of <u>maximizing processor utilization</u> and <u>minimizing communication costs</u>.** **<u>+ load balance</u>**
- **Mapping can be specified <u>statically</u> or determined at runtime by load-balancing algorithms (<u>dynamic scheduling</u>).**
  done by user program or system
- **<u>Two aspects of mapping:</u>**
  1 – **<u>Which processes will run on the same processor</u>, if necessary**
  2 – **<u>Which process runs on which particular processor</u>**
  - **mapping to a network topology/account for NUMA**

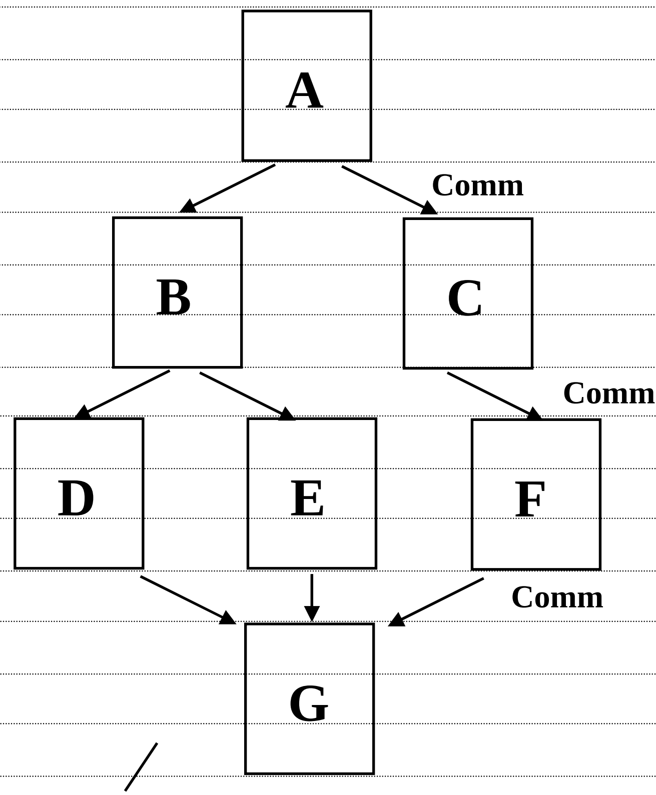
    To reduce communication time
- **One extreme: *space-sharing***
  - **Machine divided into subsets, only one app at a time in a subset**
  - **Processes can be pinned to processors, or left to OS.**
- **Another extreme: complete resource management control to OS**
  - **OS uses the performance techniques we will discuss later.**
- **Real world is between the two.**
  - **User specifies desires in some aspects, system may ignore**

<u>Task Duplication:</u> Mapping may also involve duplicating tasks to reduce communication costs

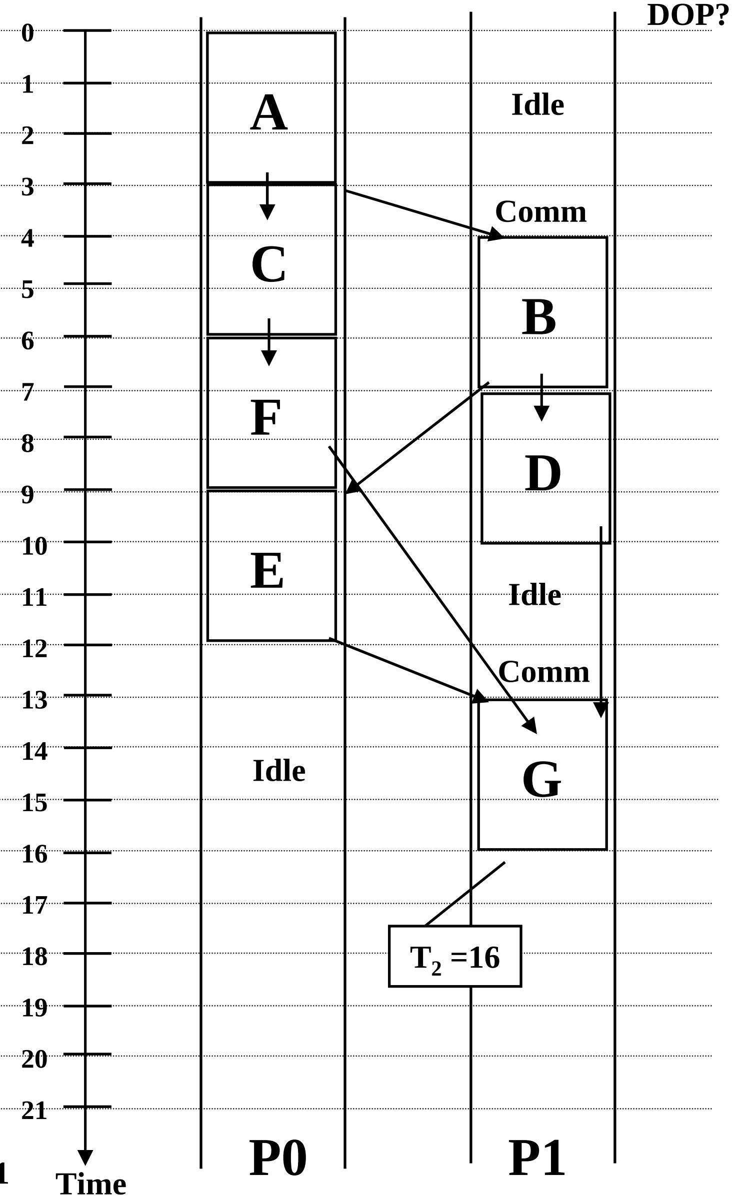
Program Partitioning Example

Example 2.4 page 64
Fig 2.6 page 65
Fig 2.7 page 66
In Advanced Computer Architecture, Hwang
(see handout)





Partitioning into tasks (given)



<u>Mapping of tasks to processors (given):</u>

P0: Tasks A, C, E, F

P1: Tasks B, D, G

Assume computation time for each task A-G = 3
Assume communication time between parallel tasks = 1
Assume communication can overlap with computation
Speedup on two processors = T1/T2 = 21/16 = 1.3

A simple parallel execution example

From lecture #1

Static Multiprocessor Scheduling

Dynamic multiprocessor scheduling is an NP-hard problem.

Node Duplication: to eliminate idle time and communication delays, some nodes may be duplicated in more than one processor.

Fig. 2.8 page 67

Example: 2.5 page 68
In Advanced Computer Architecture, Hwang
(see handout)

Table 2.1 Steps in the Parallelization Process and Their Goals

Step	Architecture-Dependent?	Major Performance Goals
Decomposition	Mostly no	Expose enough concurrency but not too much
Assignment **Determine size and number of tasks**	Mostly no **?**	Balance workload Reduce communication volume
Orchestration **Tasks → Processes**	Yes	Reduce noninherent communication via data locality Reduce communication and synchronization cost as seen by the processor Reduce serialization at shared resources Schedule tasks to satisfy dependences early
+ Scheduling Mapping **Processes → Processors** **+ Execution Order (scheduling)**	Yes	Put related processes on the same processor if necessary Exploit locality in network topology