Module IV

N-D Push Down Automata (NPDA)

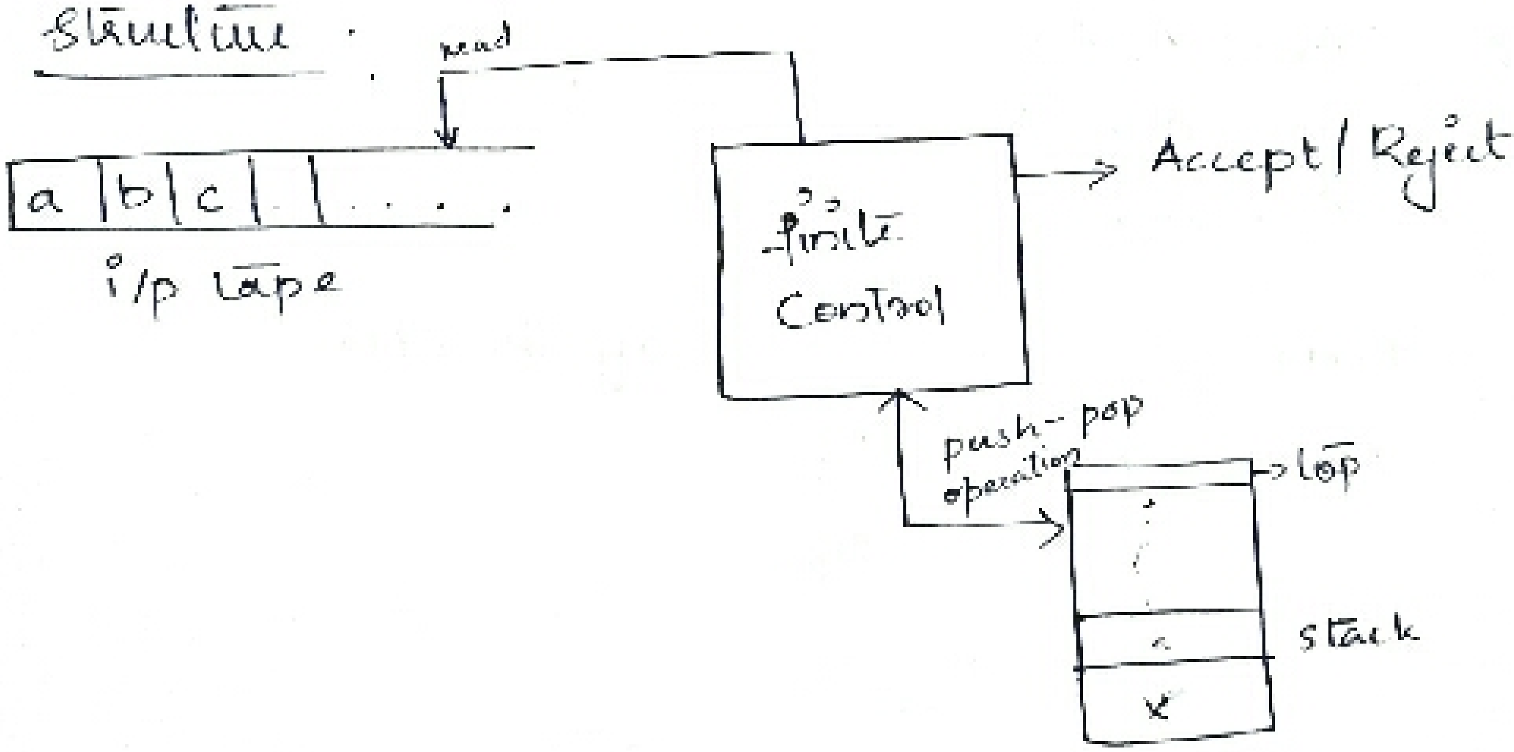
context free language $\xrightarrow{\text{acceptor}}$ PDA.

PDA = finite state machine + stack

Components of NPDA.

1. finite input tape
2. finite control
3. Stack.

Structure.



Formal Definition of NPDA. • 7 tuple notation.

$$M = (Q, \Sigma, \underset{\substack{\downarrow \\ \text{capital gamma}}}{\Gamma}, \delta, q_0, z_0, F)$$

Q – finite ~~state~~ set of states

Σ – set of input symbols.

Γ – set of stack symbols.

q_0 – initial state (in Q)

Z_0 – start ~~start~~ stack symbol (in Γ)

F – finite set of final states (in Q)

δ – $\delta : Q \times \Sigma \cup \{\epsilon\} \times \Gamma \to \cancel{Q \times \Gamma^*}\ 2^{Q \times \Gamma^*}$

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2) $A_1 \to A_2 A_3$

$A_2 \to A_3 A_1 \mid b$ convert into GNF

$A_3 \to A_1 A_2 \mid a$

Given grammar is already in CNF

Replace A_1 with D_1

" A_2 " D_2

" A_3 " D_3

$D_1 \to D_2 D_3$

$D_2 \to D_3 D_1 \mid b$

$D_3 \to D_1 D_2 \mid a$

But $D_3 \to D_1 D_2 \mid a$

substitute D_1 in D_3

$D_3 \to D_2 D_3 D_2 \mid a$.

substitute D_2 in D_3

$D_3 \to D_3 D_1 D_3 D_2 \mid b D_3 D_2 \mid a$.

avoid left recursion.

$D_3 \to \underbrace{D_3}_{A} \underbrace{D_1 D_3 D_2}_{\alpha} \mid \underbrace{b D_3 D_2 \mid a}_{\beta}$.

$D_3 \to b D_3 D_2 B \mid a B \mid b D_3 D_2 \mid a$

$B \to D_1 D_3 D_2 B \mid D_1 D_3 D_2$.

$D_1 \to D_2 D_3$

$D_1 \to D_3 D_1 D_3 \mid b D_3$.

$D_1 \to b D_3 D_2 B D_1 D_3 \mid a B D_1 D_3 \mid b D_3 D_2 D_1 D_3 \mid$

$a D_1 D_3 \mid b D_3$.

$D_2 \to b D_3 D_2 B D_1 \mid a B D_1 \mid b D_3 D_2 D_1 \mid a D_1 \mid b$.

$D_3 \to b D_3 D_2 B \mid a B \mid b D_3 D_2 \mid a$.

$B \to b D_3 D_2 B D_1 D_3 D_3 D_2 B \mid a B D_1 D_3 D_3 D_2 B \mid$

$b D_3 D_2 D_1 D_3 D_3 D_2 B \mid a D_1 D_3 D_3 D_2 B \mid b D_3 D_3 D_2 B \mid$

$b D_3 D_2 B D_1 D_3 D_3 D_2 \mid a B D_1 D_3 D_3 D_2 \mid b D_3 D_2 D_1 D_3 D_3 D_2 \mid$

$a D_1 D_3 D_3 D_2 \mid b D_3 D_3 D_2$.

3) $S \to AaCb \mid ABa$

$A \to bAa \mid a$

$B \to BaB \mid b$

$C \to c$

∴ no ϵ production, no unit production, no useless symbol.

∴ It is simplified.

① $S \to AaCb$

$S \to AD_1$

$D_1 \to D_2 D_3$

$D_2 \to a$

$D_3 \to CD_4$

$D_4 \to b$

② $S \to ABa$

$S \to AD_5$

$D_5 \to \cancel{D_6 D_2}\ BD_2$

$\cancel{D_6 \to B}$ $\cancel{D_6 \to 2}$

$D_5 \to Ba$.

③ $A \to bAa$

$A \to D_4 D_7$

$D_7 \to AD_2$

④ $B \to BaB$.

$B \to B D_8$

$D_8 \to D_2 B$

$S \to AD_1 \mid AD_5$

$A \to D_4 D_7 \mid a$

$B \to BD_8 \mid b$

$C \to c$

$D_1 \to D_2 D_3$

$D_2 \to a$

$D_3 \to CD_4$

$D_4 \to b$

$D_5 \to \cancel{D_6 D_2}\ BD_2$

$\cancel{D_6 \to B}$

$D_7 \to AD_2$

$D_8 \to \cancel{D_2 D_6}\ D_2 B$.

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δ takes as argument $\delta(q, a, X)$ where

- q is a state
- a is either an i/p symbol in Σ or $a = \epsilon$ (empty string)
- X is a stack symbol which is a member of Γ

Output of δ is a finite set of pairs (P, α) where

- P is the new state and α is the string of stack symbol that replaces X at the top of the stack.

For instance, if $\alpha = \epsilon$, then stack is popped.

If $\alpha = X$, then the stack is unchanged.

If $\alpha = YZ$, then X is replaced by Z and Y is pushed onto the stack.

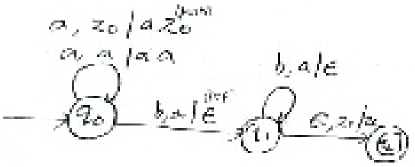
* Always stack top is leftend.

A PDA P to be deterministic (DPDA) if and only if the following conditions are met.

1. $\delta(q, a, X)$ has atmost one member for any q in Q, a in Σ or $a = \epsilon$ and X in Γ.
2. If $\delta(q, a, X)$ is non-empty for some a in Σ then $\delta(q, \epsilon, X)$ must be empty.

1) Design a (PDA) PDA to accept the language
$L = \{a^n b^n \mid n \geq 1\}$

let $s = aaabbb$



2 types of acceptance :-
Acceptance by
1) final state -

empty stack

2) empty ~~stack~~ stack

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Transitions

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$ push

$\delta(q_0, b, a) = (q_1, \epsilon)$ pop

$\delta(q_1, b, a) = (q_1, \epsilon)$

String is accepted by final state

$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

— OR

if ~~it~~ it is accepted by empty stack

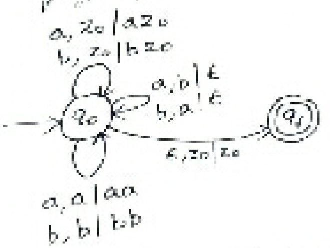
$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

If string is accepted by both,

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$

2) L = set of all strings containing equal no. of a's & b's.

$L = \{ab, ba, baba, abba, \ldots\}$

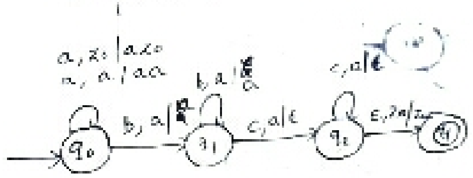


empty stack

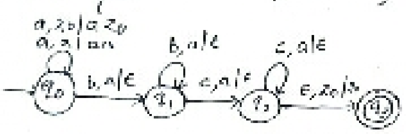
acceptance by final state.

3) $L = \{a^n b^m c^n \mid n, m \geq 1\}$

$L = \{abc, aabcc, \ldots\}$



4) $L = \{ a^{m+n} b^{m} c^{n} \mid n, m \geq 1 \}$



$Q = \{ q_0, q_1, q_2, q_3 \}$

$\Sigma = \{ a, b, c \}$

$\Gamma = \{ z_0, a, \}$

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5) $L = \{ a^{n} b^{2n} \mid n \geq 1 \}$

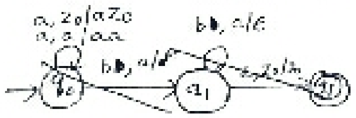
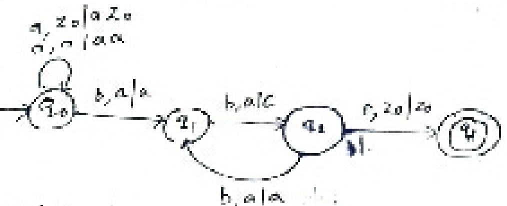


abb
aabbbb



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Instantaneous Description (ID)

ID is used formly describe the configuration of a PDA at a given instant. We define ID to be triple (q, w, α) where

1. q is the state
2. w is the remaining input
3. α is the stack contents.

Conventionly, we show the top of the stack at the left end of α and the bottom at the right end.

If $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ is a PDA then we say

$$(q, aw, z\alpha) \vdash_{M} (p, w, \beta\alpha)$$

if $\delta(q, a, z)$ contains (p, β).

1) Trace your PDA for the language $L = a^{n} b^{2n}$ $n \geq 1$ with $n = 2$

$L(M) = a^2 b^4 = aabbbb$.

initialconfig.

$(q_0, aabbbb, z_0) \vdash (q_0, abbbb, az_0) \vdash$

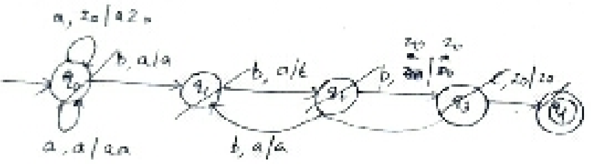
$(q_0, bbbb, aaz_0) \vdash (q_1, bbb, aaz_0) \vdash$

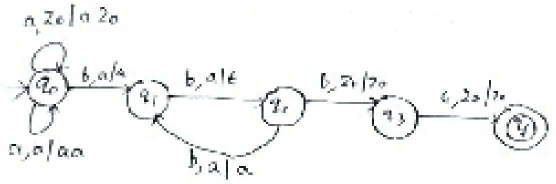
$(q_2, bb, az_0) \vdash (q_1, b, az_0) \vdash (q_2, \epsilon, z_0)$

$\vdash (q_f, \epsilon, z_0)$

2) $L = \{ a^n b^{2n+1} \mid n \geq 1 \}$

$L = \{ abbb, aabbbbb, \ldots \}$





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3) $L = \{ a^m b^{m+n} c^n \mid n, m \geq 1 \}$

Language of a PDA.

1. Acceptance by final state

Let $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ be a PDA. Then $L(P)$, the language accepted by P by final state, is

$$L(P) = \{ w \mid (q_0, w, Z_0) \vdash_P^* (q, \epsilon, \alpha) \}$$

OR

Some state q in F and any stack string α

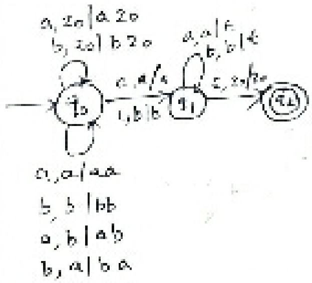
ie, starting in the initial ID with w waiting on the input, P consumes w from the input and enters an accepting state, the contents of the stack at that time is irrelevant.

2. Acceptance by empty stack.

For the PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$, we define $N(P) = \{ w \mid (q_0, w, Z_0) \vdash_P^* (q, \epsilon, \epsilon) \}$ for any state q.

ie, N(P) is the set of input w that P can consume and at the same time empty its stack.

1. Design a PDA to accept the language $L = \{ wcw^R \mid w \in (a,b)^*, w^R$ - reverse of w $\}$



$wcw^R \rightarrow$ Deter. PDA $\rightarrow$ DCFL

$ww^R \rightarrow$ Non-D PDA $\rightarrow$ NCFL

ww^R - even palind

wxw^R - odd palindrome

explain

CFL → PDA

PDA → DPDA → DCFL (wcw^R)

PDA → NPDA → NCFL (ww^R)

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1) Design a PDA for $L = \{ww^R$