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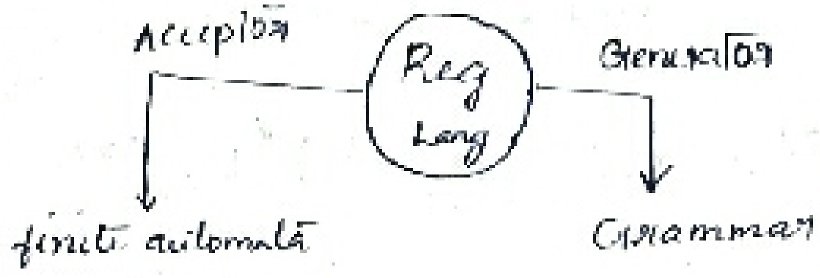
Module - 3

Grammar $(\underset{N}{V}, \underset{\Sigma}{T}, P, S)$



(V, T, P, S)

$V \rightarrow$ variables / Non-~~termina~~ terminals

$T \rightarrow$ terminals

$P \rightarrow$ production rule

$S \rightarrow$ start symbol

Eg:- $\Sigma = \{a, b\}$

$T \rightarrow \{a, b\}$

start symbol.

$\underbrace{S \rightarrow aSB}_{P}$

$S \rightarrow aB$

$B \rightarrow b$

$V \rightarrow$ capital letters $\rightarrow$ variables

Derivation $\rightarrow$ generating a string using particular grammar.

Eg:- to derive aabb

$S \Rightarrow aSB$ — sentential form. – derivation from the start symb of produ strings that m a special rule.

$\Rightarrow aaBB$

$\Rightarrow aabB$ — left most derivation

$\Rightarrow aabb$

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Eg:- $\left.\begin{array}{l} S \rightarrow aSb \\ S \rightarrow ab \end{array}\right\}$ Productions rule.

to derive

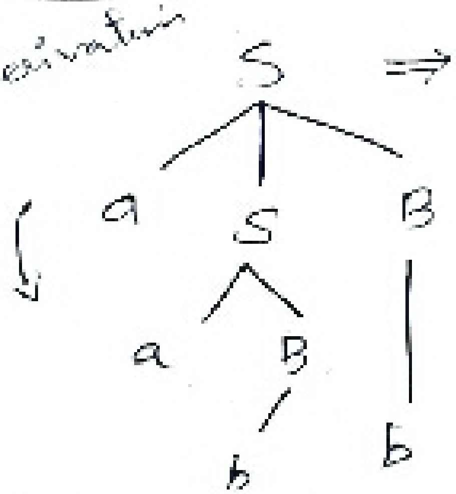
$\left.\begin{array}{l} S \Rightarrow aSb \\ \Rightarrow aabb \end{array}\right\}$ process is called derivations

previous

$\left.\begin{array}{l} S \Rightarrow aSB \\ \Rightarrow aSb \\ \Rightarrow aabb \end{array}\right\}$ right most derivations

<u>tree rep.</u>

left derivation tree



$\Rightarrow$ derivation tree / parse tree

process of generating a string using start symbol

root – start symbol of grammar

internal nodes – variable

leaf – terminals

yield $\rightarrow$ string

Pumping Lemma for Regular Language.

Let L be a regular language. Then there exist a constant N (which depends on L) such that for every string w in L such that ~~and~~ $|w| \geq N$, we can break w into 3 strings,

$w = xyz$ such that

1. $y \neq \epsilon$ or $|y| > 0$
2. $|xy| \leq n$

3. For all $i \geq 0$, the string xy^iz is also in L

ie, we can always find a non-empty string y that can be pumped ie, repeating y any no. of times or deleting it (in the case $i=0$) keeps the resulting string in the language L.

1) Prove that the language $L = \{0^n 1^n \mid n \geq 1\}$ is not regular.

Assume the given language L is regular.

$n = 3$ (Assume)

$\omega = 0^3 1^3$

$= 000111$

$|y| > 0 \Rightarrow |y| = 1$

$|xy| \leq n \Rightarrow |xy| = 3$.

$x = 0^2$

$y = 0$

$z = 1^3$

$\omega = \underset{x}{00}\underset{y}{0}\underset{z}{111}$

Now we have to prove that all $i \geq 0$, $xy^iz \in L$.

when $i = 0$.

$xy^0z = 0^2 1^3 \notin L$

$\therefore$ they doesn't satisfies pumping lemma.

2. language $L = \{a^n b^n \mid n \geq 0\}$ is not regular.

Assume the given language is regular.

$n = 3$

$|y| > 0 \Rightarrow |y| = 1$

$|xy| \leq n \Rightarrow |xy| = 3$.

$\omega = \underset{x}{aa}\underset{y}{a}\underset{z}{bbb}$

Now we have to prove that all $i \geq 0$, $xy^iz \in L$

when $i = 0$.

$xy^0z = aabbb = a^2b^3 \notin L$.

$\therefore$ they doesn't satisfies pumping lemma.

3. $L = \{a^n b a^n \mid n \geq 0\}$.

Assume the given language is regular.

$n = 2$

$|y| > 0 \Rightarrow |y| = 1$

$|xy| \leq 2 \Rightarrow |xy| = 2$.

$\omega = \underset{x}{a}\underset{y}{a}\underset{z}{baa}$

Now we have to prove that all $i \geq 0$, $xy^iz \in L$

when $i = 0$,

$xy^0z = abaa \notin L$

$\therefore$ doesn't satisfies pumping lemma.

OR

in general,

5) $L = \{a^n b a^n \mid n \geq 0\}$

~~the~~ Assume that the ~~g~~ given language is regular for a constant n, i.e. the language satisfies the conditions for pumping lemma with constant ~~m~~ n. Let $w = a^n b a^n$,

$|w| \geq n$

$|w| = 2n+1$

$|y| > 0$, $|xy| \leq n$.

Assume $x = a^i$ $y = a^j$ $\quad \underbrace{aa \ldots a}_{xy}\ \underbrace{ba \ldots a}_{z}$

$i + j \leq n$. $\quad j = n - j$

$xy^k z \in L$ for $k \geq 0$.

$k = 0$

$xy^0z = a^{i0}\ a^{n-j}\, b a^n \notin L$

$\therefore$ doesn't satisfies.

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Q1) Using pumping lemma prove that following languages are not regular.

a) $L = \{a^p \mid p \text{ is a prime no}\}$, $\Sigma = \{a\}$

b) $L = \{wcw^R \mid w \text{ is a string and } w^R \text{ is its reverse}\}$ over $\Sigma = \{a, b\}$

c) $L = \{0^n 1^{2n} \mid n \geq 1\}$

Closure properties of Regular set / Languages.

1. Union

If L_1 & L_2 are regular languages,

$L_1 \cup L_2 \Rightarrow$ regular

Eg:- $L_1 = \{0, 000, 00000, \ldots\}$ $0(00)^*$

$L_2 = \{\epsilon, 00, 0000, \ldots\}$ $(00)^*$

$L_1 \cup L_2 = \{\epsilon, 0, 00, 000, \ldots\} = 0^* \Rightarrow$ regular

2. Intersection.

If L_1 & L_2 are regular

$L_1 \cap L_2 \Rightarrow$ regular.

Eg:- $L_1 = \{$set of all strings over $\Sigma = \{0\}\}$

$L_2 =$ set of all strings of even length over $\Sigma = \{0\}$

$L_1 \cap L_2 =$ set of all strings of even length over $\Sigma = \{0\}$

3. Concatenation.

If L_1 & L_2 - regular
$L_1 . L_2 \Rightarrow$ Regular.

Eg:- $L_1 = \{ a^m \mid m \geq 0 \}$

$L_2 = \{ b^n \mid n \geq 0 \}$

$L_1 L_2 = \{ a^m b^n \mid m,n \geq 0 \}$

4. Kleene closure

If L is regular then L^* is also regular.

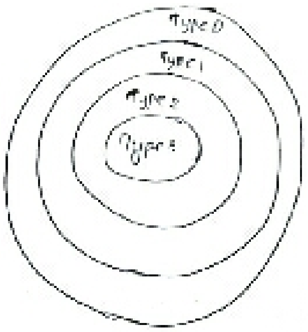
$L = \{0, 10\} \Rightarrow (0+10)$

$L^* = \{ 0, 000, 10, 10010, 010, 100, 1000, \ldots \}$
$= (0+10)^*$

Chomsky Hierarchy.

In TOC, Grammar is divided as

1) Type 0 → Recursively enumerable Grammar Language
2) " 1 → Context Sensitive Language.
3) " 2 → Context free Language
4) " 3 → Regular Language
(Regular Grammar)



HW

1)
a) Assume the given ... language L is regular

$n = 3$

$w = a^3$
$= aaa$

$|y| > 0 \Rightarrow |y| = 1$

$|xy| \leq n \Rightarrow |xy| = 3$

$w = \underset{x}{a} \, \underset{y}{a} \, a$

Now we have to prove that all $i \geq 0$, $xy^iz \in L$

when $i = 0$

$xy^iz = a^2 \notin L$.

∴ they doesn't satisfies pumping lemma.

b) Assume the given language L is regular.

$n=4$

$w = abcba$

$|y|>0 \Rightarrow |y|=1$

$|xy| \le n \Rightarrow |xy| = 4$

$w = \underbrace{abc}_{x}\underbrace{b}_{y}\underbrace{a}_{z}$

Now we have to prove that all $i \ge 0$, $xy^iz \in L$

when $i=0$

$xy^0z = abca \notin L$

∴ they doesn't satisfies pumping lemma.

c) $L = \{0^n 1^{2n} \mid n \ge 1\}$

Assume the given language L is regular.

$n=4$

$w = 0^4 1^8$

$w = 000011111111$

$|y|>0 \Rightarrow |y|=1$

$|xy| \le n \Rightarrow |xy| = 4$

$w = \underbrace{000}_{x}\underbrace{0}_{y}\underbrace{11111111}_{z}$

Now we have to prove that all $i \ge 0$, $xy^iz \in L$.

when $i=0$,

$xy^0z = 00011111111 \notin L$

∴ they doesn't satisfies pumping lemma.

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1) Consider the following grammar (even-length string)

$S \rightarrow AS \mid \epsilon$

$A \rightarrow aa \mid ab \mid ba \mid bb$.

Give leftmost derivation, rightmost derivation and one parse tree for the string.

aabbba

Leftmost

$S \rightarrow AS$ ($S \rightarrow AS \mid \epsilon$)

$\Rightarrow aaS$ ($A \rightarrow aa$)

$\Rightarrow aaAS$ ($S \rightarrow AS$)

$\Rightarrow aabbS$ ($A \rightarrow bb$)

$\Rightarrow aabbAS$ ($S \rightarrow AS$)

$\Rightarrow aabbbaS$ ($A \rightarrow ba$)

$\Rightarrow aabbba\epsilon$ ($S \rightarrow \epsilon$)

$\Rightarrow$ aabbba

Rightmost

$S \rightarrow AS$

$\Rightarrow AAS$ (By $S \rightarrow AS$)

$\Rightarrow AAAS$ (By $S \rightarrow AS$)

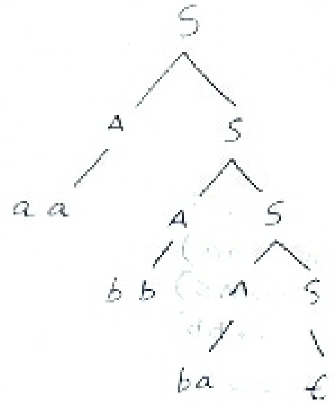
$\Rightarrow AAA\epsilon$ (By $S \to \epsilon$)

$\Rightarrow AAba$ ($A \to ba$)

$\Rightarrow A\,bb\,ba$ ($A \to bb$)

$\Rightarrow aabbba$ ($A \to aa$)

Leftmost derivation tree.



H.W.

2) Check whether the grammar

$S \to 0B \mid 1A$

$A \to 0 \mid 0S \mid 1AA \mid \epsilon$

$B \to 1 \mid 1S \mid 0BB$

generates the string 00110101

Context free - grammar

Production rules $\Rightarrow$ $A \to \alpha$

$A \in V$, $\alpha \in (V \cup T)^*$

Construct context-free grammar.

1. Language of any no. of a's (R.E - a^*)

$T = \{a, \epsilon\}$ $S = \{S\}$

$L = \{\epsilon, a, aa, \dots\}$ $V = \{S\}$

$\therefore$ $S \to aS / \epsilon$

2. $L = \{aa, ab, ba, bb\}$ (R.E - $(a+b)(a+b)$)

$T = \{a, b, \epsilon\}$

$S \to AA$

$A \to a / b$

3. $a(a^* b^*)b$

$L = \{ab, aabb, aaab, \dots\}$

$S \to aMb$

$M \to AB$

$A \to aA / \epsilon$

$B \to bB / \epsilon$

4) $(a+b)^*$

$S \to aS \mid bS \mid \epsilon$

5) $a\,(a+b)^*\,b$

$S \to aMb$

$M \to aM \mid bM \mid \epsilon$

6) $(a+b)(a+b)(a+b)^*$ {atleast 2-length}

$S \to AAM$

$A \to a / b$

$M \to aM \mid bM \mid \epsilon$

7) a^+

$S \to aS / a$

8) $L = \{a^n b^m / n,m \geq 1\}$

$L = \{ab, abb, aab, \dots\}$

$S \to AB$

$A \to aA \mid a$

$B \to bB \mid b$

9) $L = \{a^n b^n / n \geq 1\}$

$L = \{\underline{ab}, \underline{aabb} \dots\}$

$S \to aSb / ab$

1.2)

$S \Rightarrow 0B$

$\Rightarrow 00BB$ (by $B \to 0BB$)

$\Rightarrow 001SB$ (by $B \to 1S$)

$\Rightarrow 0011AB$ (by $S \to 1A$)

$\Rightarrow 00110SB$ (by $A \to 0S$)

$\Rightarrow 001101AB$ (by $S \to 1A$)

$\Rightarrow 0011010B$ (by $A \to 0$)

$\Rightarrow \underline{\underline{00110101}}$ ($B \to 1$)

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10) $\{a^n b^{2n} \mid n \geq 1\}$ $L = \{abb, aabbbb \dots\}$

$S \to aSbb / abb$

11) $L = \{a^n b^n c^m \mid n,m \geq 1\}$

$\cancel{S \to aSbSc / ab}$

$S \to AB$

$A \to aAb \mid ab$

$B \to cB \mid c$

12) $L = \{\underbrace{a^n b^n}_{a} \underbrace{c^m d^m}_{b} \mid n,m \geq 1\}$

$S \to AB$

$A \to aAb \mid ab$

$B \to cBd \mid cd$

13) $L = \{a^n b^m c^n ; n, m \geq 1\}$

$L = \{abc, aabcc, \ldots\}$

$S \to aSc \mid aAc$

$A \to bA \mid b$

Imp

14) $L = \{WcW^R \mid W \in \{a,b\}^*\}$ W^R - reverse of W.

$L = \{abcba, \ldots\}$

$S \to aSa \mid bSb$

$S \to c$

Imp

15) $L = \{WW^R\}$ → palindrome even.

$L = \{aa, bb, abba, \ldots\}$

$S \to aSa \mid bSb \mid \epsilon$

* All palindrome.

$L = aba\underline{b}aba$

$S \to aSa \mid bSb \mid a \mid b \mid \epsilon$

* odd palindrome

$S \to aSa \mid bSb \mid a \mid b$

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Ambiguity - Every grammar for the language has more than one structure in some string in the language.

→ more than one left or right most derivation occurs.

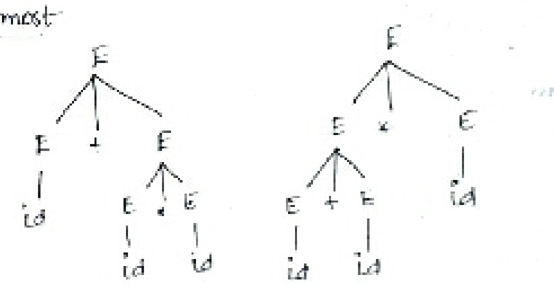
Eg:- $E \to E+E \mid E*E$

$E \to id$

$T = \{id, *, +\}$

Sample string - $id + id * id$

Leftmost



Simplification of Context free Grammar.

Optimization of context free grammar.

Steps

1. Elimination of useless symbols
2. Elimination of null / ϵ production
3. Elimination of unit production.

1. Elimination of useless symbols

(i) Identify the non-generating symbol and eliminate those productions containing them.

(ii) Identify non-reachable symbols and eliminate those productions containing them.

$L(G) \rightarrow$ Language of Grammar

$L(G) = \{ w \mid w \in T^* \text{ and } S \overset{*}{\Rightarrow} w \}$

1) Simplify the grammar

$S \rightarrow AB \mid a$
$A \rightarrow b$

Eliminate useless symbols

(i) Eliminate non-generating symbols

B is non-generating

remove B, ~~...~~ $S \rightarrow AB$

After, $S \rightarrow a$
$A \rightarrow b$

(ii) Eliminate non-reachable ~~sym~~ symbols

A is non-reachable.

remove A

After $S \rightarrow a$

2) $S \rightarrow aB \mid bX$
$A \rightarrow BAd \mid bSX \mid a$
$B \rightarrow aSB \mid bBX$
$X \rightarrow SBD \mid aBX \mid ad$

(i) B, D is non-generating symbol

remove B, D

$S \rightarrow bX$
$A \rightarrow bSX \mid a$
$X \rightarrow ad$

(ii) A is non-reachable

$S \rightarrow bX$
$X \rightarrow ad$

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3) $S \rightarrow aAa \mid aBC$
$A \rightarrow aS \mid bD$
$B \rightarrow aBa \mid b$
$C \rightarrow abb \mid DD$
$D \rightarrow aDA$

(i) Eliminate non-generating symbols.

generating variables = (B,C) (S)
$T = \{a, b, B, C, S, A\}$

D is non-generating.

$S \rightarrow aAa \mid aBC$
$A \rightarrow aS$
$B \rightarrow aBa \mid b$ $C \rightarrow abb$.

(ii) eliminate non-reachable symbols.

Reachable set = {S, a, A, B, C, b}

~~∴ C is not reachable~~ ~~since a, b are already reachable.~~

~~S → aAa~~
~~A → aS~~
~~B → aBa / b.~~

no symbol is non-reachable.

∴ S → aAa / aBC
A → aS
B → aBa / b
C → abb

Q) A → xyz / Xyzz
X → Xz / aYz
Y → yYy / Xz
Z → Zy / z.

(i) eliminate non-generating symbol.

X, Y is non-generating.

generating = {x, y, z, A, Z}
prodn cont X Y → remove.

A → xyz
Z → Zy / z

(ii) eliminate non-reachable symbols

Z is non-reachable. remove

∴ A → xyz.

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2. Elimination of null/ε productions.

A → ε   A $\overset{*}{\Rightarrow}$ ε

to find nullable variables.

3. Elimination of unit production

A → B } unit production.

1) S → ABAC
A → aA | ε
B → bB | ε
C → c

Removal of null productions

A → ε, ~~B → ε~~   replace A by ε

{S → ABAC / ABC / BC / ABAC
{A → aA | a

← of possible substitution for

B → ε

S → ABAC / AAC / BAC / ABC / BC / AC
B → bB | b
A → aA | a
C → c

Removal of unit production

In general
$A \rightarrow B$
$B \rightarrow \alpha$
$A \rightarrow \alpha$

$S \rightarrow C \quad \{C \rightarrow c\}$

$S \rightarrow c$

$S \rightarrow ABAC / BAC / ABC / AAC / BC / AC / c$

$A \rightarrow aA / a$

$B \rightarrow bB / b$

$C \rightarrow c$

Q) $S \rightarrow AB$

$A \rightarrow aAA / \epsilon$

$B \rightarrow bBB / \epsilon$

1. Removal of null production.

$A \rightarrow \epsilon, \; B \rightarrow \epsilon$

$S \rightarrow AB / B / A / \epsilon$

$A \rightarrow aAA / aA / a$

$B \rightarrow bBB / bB / b$

2. Removal of unit production.

$S \rightarrow B \qquad S \rightarrow A$

$S \rightarrow AB / bBB / bB / b / aAA / aA / a / \epsilon$

$A \rightarrow aAA / aA / a$

$B \rightarrow bBB / bB / b$

H.W.

1) $S \rightarrow a / Xb / aYa$

$X \rightarrow Y / \epsilon$

$Y \rightarrow b / X$.

Normal forms.

1) CNF (Chomsky Normal form)

2) GNF (Greibach Normal form)

1. CNF

A context free grammar is in CNF if all the productions are of the form $A \rightarrow BC$ or $A \rightarrow a$ where $A, B, C \in V$, $a \in T$

Q) $S \rightarrow AAC$

$A \rightarrow aAb / \epsilon$

$C \rightarrow aC / a$

Steps
1. Simplified CFG
2. Convert to CNF/GNF

1. Removal of null production.

$A \rightarrow \epsilon$

$S \rightarrow AAC / AC / C$

$A \rightarrow aAb / ab$

$C \rightarrow aC / a$

2. Removal of unit production.

$S \rightarrow C \qquad S \rightarrow AAC / AC / aC / a$

$A \rightarrow aAb / ab$

$C \rightarrow aC / a$

3. Removal of useless symbol.

(i) elimination of non-generating symbol

all are generating

(ii) eliminate non-reachable

all are reachable.

$S \to AAC \mid AC \mid aC \mid a$

$A \to aAb \mid ab$

$C \to aC \mid a$

To convert in CNF

① $S \to AAC$

$S \to AD_1$

$D_1 \to AC$.

② $S \to aC$

$S \to D_2 C$

$D_2 \to a$

③ $A \to aAb$.

$A \to D_2 D_3$

$D_2 \to a$

$D_3 \to AD_4$

$D_4 \to b$.

④ $A \to ab$

$A \to D_2 D_4$

⑤ $C \to aC$

$C \to D_2 C$.

$S \to AD_1 \mid AC \mid D_2C \mid a$

$A \to D_2D_3 \mid D_2D_4$

$C \to D_2C \mid a$

$D_1 \to AC$

$D_2 \to a$

$D_3 \to AD_4$

$D_4 \to b$

19/10/19 Convert into CNF

Q) $S \rightarrow AB / aB$

$A \rightarrow aab / \epsilon$

$B \rightarrow bbA$

(i) Removal of null production

null $\rightarrow$ $A \rightarrow \epsilon$

$S \rightarrow AB \mid aB \mid B$

$A \rightarrow aab$

$B \rightarrow bbA / bb$.

(ii) Removal of unit productions

$S \rightarrow B$

$S \rightarrow AB / aB / bbA / bb$

$A \rightarrow aab$

$B \rightarrow bbA / bb$

(iii) Removal of useless symbols.

(i) Eliminate non-generating symbol.

all are generating

(ii) Eliminate non-reachable symbol.

all are reachable.

$\therefore$ $S \rightarrow AB / aB / bbA / bb$

$A \rightarrow aab$

$B \rightarrow bbA / bb$

To convert in CNF

Format :- $A \rightarrow BC$ or $A \rightarrow a$.

(1) $S \rightarrow aB$.

$S \rightarrow D_1 B$

$D_1 \rightarrow a$.

(2) $S \rightarrow bbA$

$S \rightarrow D_2 D_3$

$D_2 \rightarrow b$

$D_3 \rightarrow bA$ $D_2 A$

(3) $S \rightarrow bb$

$S \rightarrow D_2 D_2$

(4) $A \rightarrow aab$.

$A \rightarrow D_1 D_5$

$D_5 \rightarrow ab \rightarrow D_1 D_2$

(5). $B \rightarrow bbA$

$B \rightarrow D_2 D_3$

(6). $B \rightarrow bb$

$B \rightarrow D_2 D_2$.

Grammar in CNF :-

$S \rightarrow AB \mid D_1 B \mid D_2 D_3 \mid D_2 D_2$

$A \rightarrow D_1 D_5$

$B \rightarrow D_2 D_3 / D_2 D_2$

$D_1 \rightarrow a$.

$D_2 \rightarrow b$

$D_3 \rightarrow D_2 A$

$D_5 \rightarrow D_1 D_2$

1) $S \rightarrow aSb \mid ab$. into CNF

GNF

A context free grammar is in GNF, if all the productions are of the form $A \rightarrow a\alpha$ or $A \rightarrow a$ where a is a terminal and $\alpha \in V^*$ and can be empty.

convert if it is CNF → GNF

1) $S \rightarrow AA \mid b$
$A \rightarrow SS \mid a$
convert to GNF.

① given grammar is in CNF.
② Number the variables. Replace S with D_1
Replace A with D_2.

$D_1 \rightarrow D_2 D_2 \mid b$
$D_2 \rightarrow D_1 D_1 \mid a$

③ check not left right — no
longest
production
$D_2 \rightarrow D_1$

④
$D_2 \rightarrow D_1 D_1 \mid a$

Substitute $D_1 \rightarrow D_2 D_2 b$ D_1 in D_2

$D_2 \rightarrow D_2 D_2 D_1 \mid bD_1 \mid a$

$A_i \rightarrow A_j A_n \ldots A_n$
$i < j$.
if $i = j$
left recursion

$D_1 \rightarrow D_2 D_2 \mid b$
$D_2 \rightarrow D_2 D_2 D_1 \mid bD_1 \mid a$

H.W

1) $S \rightarrow aSb \mid ab$
Simplification is not needed.
∴ convert into CNF
$A \rightarrow BC$ $A \rightarrow a$.

① $S \rightarrow aSb$
$S \rightarrow D_1 D_2$
$D_1 \rightarrow a$
$D_2 \rightarrow SD_3$
$D_3 \rightarrow b$

② $S \rightarrow ab$
$S \rightarrow D_1 D_3$.

$S \rightarrow D_1 D_2 \mid D_1 D_3$
$D_1 \rightarrow a$
$D_2 \rightarrow SD_3$
$D_3 \rightarrow b$

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⑤ Avoid left recursion.
to avoid left recursion :-
$A \rightarrow A\alpha \mid \gamma$ → productions with left recursion
Replace with
$A \rightarrow \gamma B \mid \gamma$
$B \rightarrow \alpha B \mid \alpha$

or

$A \to A\alpha_1 \mid A\alpha_2 \mid A\alpha_3 \ldots \mid A\alpha_i \mid \gamma_1 \mid \gamma_2 \ldots \mid \gamma_j$

Replace with.

$A \to \gamma_1 B \mid \gamma_2 B \mid \ldots \mid \gamma_j B \mid \gamma_1 \mid \gamma_2 \ldots \mid \gamma_j$

$B \to \alpha_1 B \mid \alpha_2 B \mid \ldots \alpha_i B \mid \alpha_1 \mid \alpha_2 \ldots \mid \alpha_i$

$D_2 \to D_2 \underbrace{D_2 D_1}_{\alpha} \mid \underbrace{b D_1}_{\gamma_1} \mid \underbrace{a}_{\gamma_2}$

$D_2 \to bD_1B \mid aB \mid bD_1 \mid a$

$B \to D_2 D_1 B \mid D_2 D_1$

check whether all are in GNF

$D_1 \to bD_1BD_2 \mid aBD_2 \mid bD_1D_2 \mid aD_2 \mid b$

$D_2 \to bD_1B \mid aB \mid bD_1 \mid a$

$B \to bD_1BD_1B \mid aBD_1B \mid bD_1D_1B \mid aD_1B \mid$
$bD_1BD_1 \mid aBD_1 \mid bD_1D_1 \mid aD_1$

H.W

1) $S \to CA \mid BB$
$B \to b \mid SB$ convert into GNF.
$C \to b$
$A \to a$

Restriction of convert to Given grammar is in CNF.

Replace S with D_1
" C " D_2
" A " D_3
" B " D_4

$D_1 \to D_2 D_3 \mid D_4 D_4$

$D_4 \to b \mid D_1 D_4$

$D_2 \to b$

$D_3 \to a$

$D_4 \to b \mid D_1 D_4$

Substitute D_1 in D_4

$D_4 \to b \mid D_2 D_3 D_4 \mid D_4 D_4 D_4$

Substitute D_2 in D_4

$D_4 \to b \mid bD_3D_4 \mid D_4D_4D_4$.

avoid left recursion.

$D_4 \to \cancel{D_4 D_4 D_4} \mid \cancel{bD_3D_4 \mid b}$

$D_4 \to \underbrace{b \mid bD_3D_4}_{\gamma} \mid \underbrace{D_4}_{A} \underbrace{D_4 D_4}_{\alpha}$

$D_4 \to bB \mid bD_3D_4B \mid b \mid bD_3D_4$

$B \to D_4 D_4 B \mid D_4 D_4$

$D_1 \to bD_3 \mid bBD_4 \mid bD_3D_4BD_4 \mid bD_4 \mid bD_3D_4D_4$

$D_4 \to bB \mid bD_3D_4B \mid b \mid bD_3D_4$

$D_2 \to b$

$D_3 \to a$

$B \to \cancel{D_4 D_4}$ $bBD_4B \mid bD_3D_4BD_4B \mid bD_4B \mid bD_3D_4D_4B \mid bBD_4 \mid bD_3D_4BD_4 \mid bD_4 \mid bD_3D_4D_4$