2/9/2019

MODULE II

BOOLEAN ALGEBRA

Axioms and Laws of Boolean Algebra

Axioms

Multiplication	OR Addition	Complement
$0 \cdot 0 = 0$	$0+0 = 0$	$\bar{1} = 0$
$0 \cdot 1 = 0$	$0+1 = 1$	$\bar{0} = 1$
$1 \cdot 0 = 0$	$1+0 = 1$	
$1 \cdot 1 = 1$	$1+1 = 1$	

Logical AND

Laws of Boolean Algebra

AND Laws	OR LAWS
$x \cdot 0 = 0$	$x+0 = x$
$x \cdot 1 = x$	$x+1 = 1$
$x \cdot x = x$	$x+x = x$
$x \cdot \bar{x} = 0$	$x+\bar{x} = 1$

COMPLIMENTATION Law (Double Negation Laws)

$\bar{\bar{x}} = x$

Commutative Law

$x+y = y+x$

$x \cdot y = y \cdot x$

Associative Law

$x+(y+z) = (x+y)+z$

$x \cdot (y \cdot z) = (x \cdot y) \cdot z$

Distributive Law

$x+(y \cdot z) = (x+y) \cdot (x+z)$

$x \cdot (y+z) = x \cdot y + x \cdot z$

Idempotent Law

$x \cdot x = x$

$x+x = x$

Negation Law

$x \cdot \bar{x} = 0$

$x+\bar{x} = 1$

Null Law

$x \cdot 0 = 0$

$x+0 = x$

Absorption law:

$x + x \cdot y = x$

$x \cdot (x+y) = x$

IMP **Demorgan's Theorem**

$\overline{x+y} = \bar{x} \cdot \bar{y}$

$\overline{x \cdot y} = \bar{x} + \bar{y}$

Duality

~~AND (·) → (OR)~~

changes

AND (·) → OR (+)

OR (+) → AND (·)

0 → 1

1 → 0

Eg:-

$A \cdot 1 = A$

$A + 0 = A$

$\overline{x+y} = \bar{x} \cdot \bar{y}$

$\overline{x \cdot y} = \bar{x} + \bar{y}$

⇒ Demorganise or Apply Demorgan's Theorem to find complement or the following expression:

1. $\cancel{(A+\bar{B})}$

$(A+\bar{B}) \cdot (C+\bar{D})$

$\overline{\underset{x}{(A+\bar{B})} \cdot \underset{y}{(C+\bar{D})}} = \overline{(A+\bar{B})} + \overline{(C+\bar{D})}$

$= (\bar{A} \cdot \bar{\bar{B}}) + (\bar{C} \cdot \bar{\bar{D}})$

$= \bar{A} \cdot B + \bar{C} \cdot D$

$\overline{x+y} = \bar{x} \cdot \bar{y}$

$\overline{x \cdot y} = \bar{x} + \bar{y}$

$\bar{\bar{x}} = x$

2. $\cancel{\overline{A\bar{B} + \bar{A} + AB}} = \cancel{\bar{A} + \bar{\bar{B}}} + \cancel{A}$

$\overline{x \cdot y} = \bar{x} + \bar{y}$

$\cancel{\bar{\bar{A}}\bar{\bar{B}} + \bar{\bar{A}} + \bar{A}\bar{B}} = \cancel{AB + A + \bar{A} + \bar{B}}$

2. $\overline{A\bar{B} + \bar{A} + AB} = \overline{A\bar{B}} + \overline{\bar{A}} + \overline{AB}$

$= \bar{\bar{A}}\bar{B} \cdot \bar{\bar{A}} \cdot \overline{AB}$ Demorgan's law

$= AB \cdot A \cdot \overline{AB}$

$= AB \cdot \overline{AB}$

$= 0$

$\boxed{\begin{array}{c} AB \cdot A \\ A \cdot B \cdot A \\ A \cdot A \cdot B \\ AB \end{array}}$

3. $\cancel{F = \overline{A\bar{B} + B\bar{C} + A}}$

$F = A\bar{B} + B\bar{C} + \bar{A}C$

$= \overline{A\bar{B} + B\bar{C} + \bar{A}C}$ [Demorgan's law]

$= (\bar{A}+B) \cdot (\bar{B}+C) \cdot (A+\bar{C})$

4. $f(x,y,z) = x(y'+z)$

$= \overline{x(y'+z)} = \bar{x} + (\overline{y'+z})$

$= \bar{x} + (y \cdot \bar{z})$

⇒ Simplify the following Boolean expression or Boolean functions using laws of Boolean Algebra.

1. $x(\bar{x}+y)$ $= x\cdot\bar{x} + x\cdot y$ (Distributive law)

$= 0 + xy$ $(x\cdot\bar{x} = 0)$

$= xy$ $(0 + x = x)$

2. $AB + A(B+C) + B(B+C)$

$= \cancel{BA}^{AB} + A\cdot B + A\cdot C + B\cdot B + B\cdot C$

$= AB + A\cdot B + A\cdot C + B + B\cdot C$ (x.x

$= AB + A.(B+C) + B + B\cdot C$ (Distributive law)

$= AB + A(B+C) + B$

$= AB + AB + AC + B$ $(x+x=x)$

$= AB + AC + B$

$= AC + \overset{xy+x}{BA + B}$ (Absorption law)

$= AC + B$

3. $(x+y)(x+y')$

4. $xy + xyy'$

5. $ABC\,[AB + \bar{C}(BC+AC)]$

⇒ Prove the following Boolean identity using laws of Boolean Algebra.

$x + x'y = x+y$

LHS $= x + x'y$

$= (x+x')\cdot(x+y)$ (Distributive law)

$= 1\cdot(x+y)$

$= (x+y)$

$\therefore$ LHS = RHS

2. $AB + \bar{A}C = (A+C)(\bar{A}+B)$

3. $AB + \bar{A}C + A\bar{B}C(AB+C) = 1$

4. $\overline{AB} + \bar{A} + AB$

Logic Gates

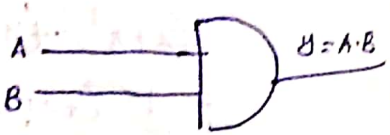
AND OR NOT

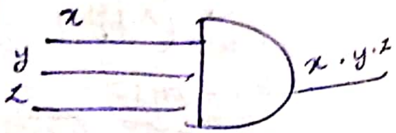
Universal gates - NAND, NOR - we can do the all basic operations AND OR NOT using universal gates

XOR, XNOR

1. AND Gate (All or nothing Gate)

i/p		o/p
A	B	$y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1





$(x \cdot y) \cdot z$

2. OR Gate (Any or all gate)

A	B	$y = A+B$
0	0	0
0	1	1
1	0	1
1	1	1

3. NOT (Inverter) $y = \bar{A}$

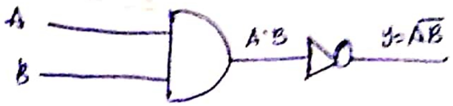
A	$y = \bar{A}$
0	1
1	0

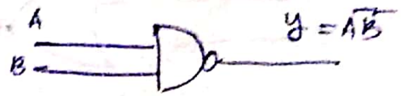
Universal Gates

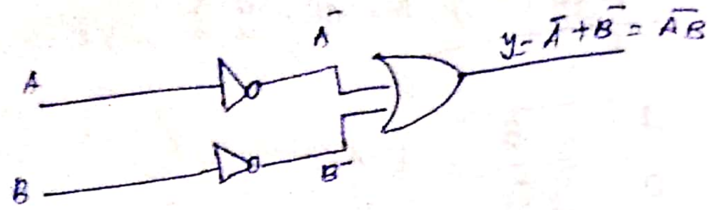
4. NAND Gate

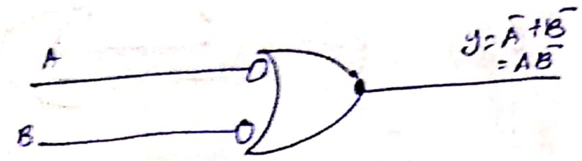
~~NAND~~ NAND $\longrightarrow$ NOT AND

A	B	$y = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0





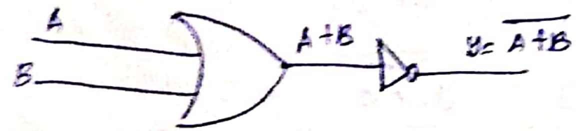




Negative OR gate / Bubbled gate

5. <u>NOR Gate</u>

A	B	$y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

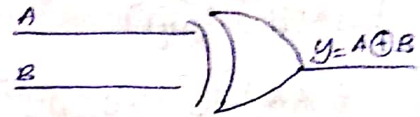


Special gates

6. XOR Gate $\Rightarrow y = A \oplus B$

$= \bar{A}B + A\bar{B}$

A	B	$y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

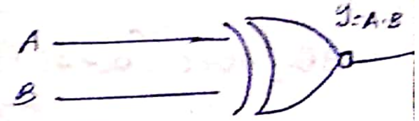


7 <u>XNOR Gate</u>

$y = A \odot B = AB + \bar{A}\bar{B}$

XOR
↓ NOT

A	B	$y = A \odot B$
0	0	1
0	1	0
1	0	0
1	1	1



$AB + \bar{A}C = \cancel{(A+C)(\bar{A}+B)}$

$AB + \bar{A}C = \cancel{BA} + \bar{A}C$

$= B$

Home work - Answers

1. $(x+y)(x+y')$

$= xx + xy' + xy + yy'$

$= x + x(y'+y) + 0$

$= x + x$

$= \underline{\underline{2x}}$

2. $xy + xyy'$

$= \cancel{xy(1+y')}\ yy' = 0$

$= \underline{\underline{xy}}$

3) $ABC\left[AB + \bar{C}\,(BC + AC)\right]$

4) $\overline{AB} + \bar{A} + AB = \bar{A} + \bar{B} + \bar{A} + B\cdot A$

$= \bar{A} + \bar{B} + \bar{A} + A\cdot B$

$= \bar{A} + \bar{B} + B$

$= \underline{\underline{\bar{A} + 1}}$

Ans ⇒ Implement AND, OR, NOT, XOR operations in terms of NAND Gates and NOR gates

OR

prove NAND and NOR gates are universal gates.

1) Using NAND Gate

i) NOT Gate

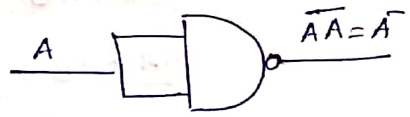
$y = \bar{A}$

for NAND Gate

$y = \overline{AB}$ let $[A = B]$

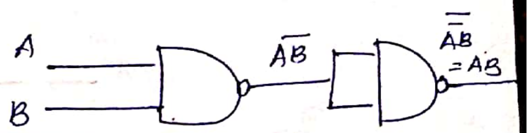
$= \overline{AA}$

$= \underline{\underline{\bar{A}}}$



ii) AND Gate

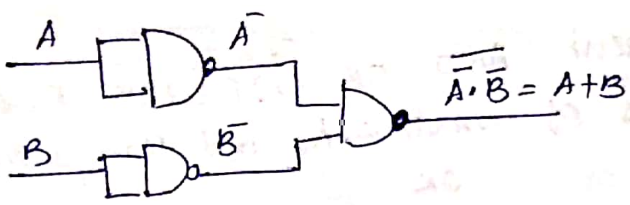
$y = AB = \overline{\overline{AB}}\}_{NAND}$



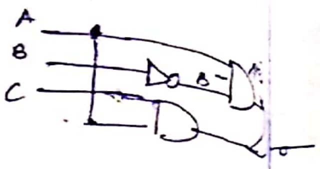
iii) OR Gate

$y = A+B$

$= \overline{\overline{A+B}} = \overline{\bar{A}\cdot\bar{B}}$ (Demorgan's law)



$A\bar{B} + CA$



III) XOR Gate

$y = A \oplus B$

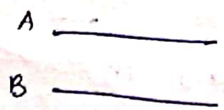
$= \bar{A}B + A\bar{B}$

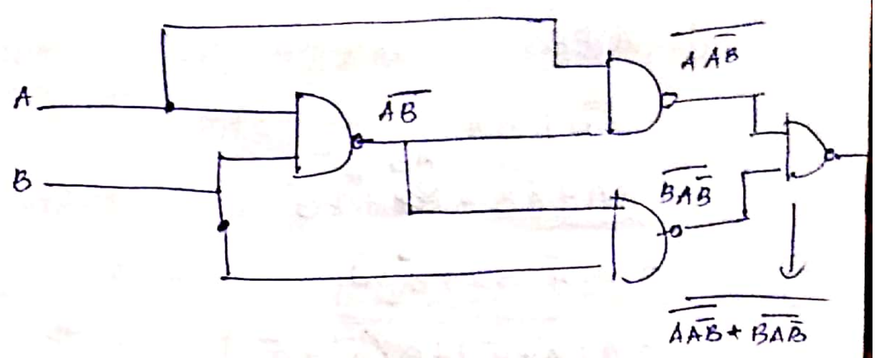
$= A\bar{A} + \bar{A}B + A\bar{B} + B\bar{B}$

$= A(\bar{A}+\bar{B}) + B(\bar{A}+\bar{B})$

$= A\overline{AB} + B\overline{AB}$

$= \overline{\overline{A\overline{AB} + B\overline{AB}}}$





$= A \oplus B$

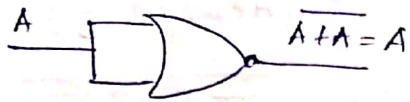
Using NOR

1) NOT Gate

$y = \overline{A+B}$ $(A=B)$

$= \overline{A+A}$

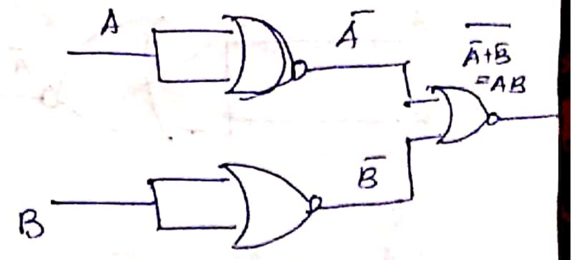
$= \bar{A}$



ii) AND Gate

$y = A \cdot B$

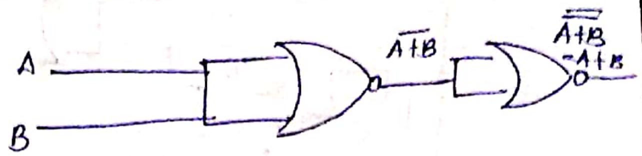
$y = \overline{\overline{AB}} = \overline{\bar{A}+\bar{B}}$ (Demorgan's)



iii) OR Gate

$y = A+B$

$= \overline{\overline{A+B}}$ ~~A~~



iv) XOR Gate

$y = A \oplus B$

$= \bar{A}B + \bar{B}A$

$= A\bar{A} + \bar{A}B + \overset{A\bar{B}}{\cancel{B\bar{A}}} + B\bar{B}$

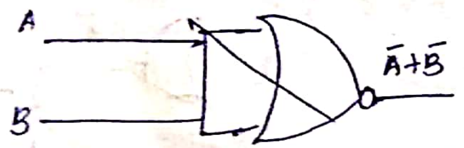
$= \cancel{A(\bar{A}+\bar{B}) + \bar{B}[A}$

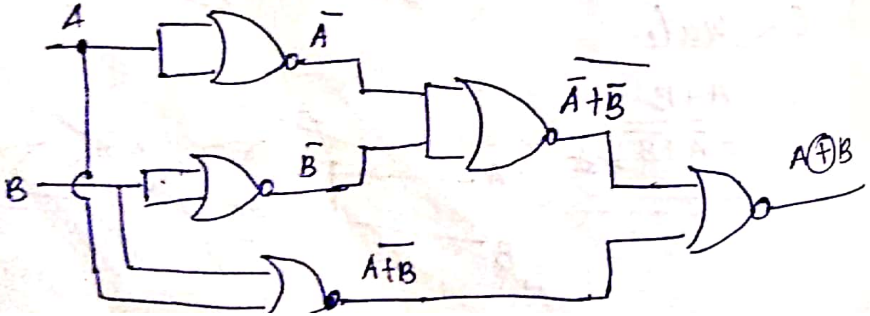
$= A(\bar{A}+\bar{B}) + B(\bar{A}+\bar{B})$

$= (A+B)(\bar{A}+\bar{B})$

$= \overline{\overline{(A+B)(\bar{A}+\bar{B})}}$

$= \overline{\overline{(A+B)} + \overline{\bar{A}+\bar{B}}}$





SOP - sum of product form

Standard SOP - $AB + BC' + A$ $\quad A'B'C'$ ABC

&

Canonical SOP - $ABC + BC'A + AB'C$

(sum of minterms)

POS - product of sum

Standard POS = $(A+B').(B+C).A$

Canonical POS = $(A+B'+C)(A'+B+C)$

(product of maxterms)

Each product terms of canonical sop is called Minterms

Each sum term of canonical POS is called Maxterms

$\Rightarrow$ $f(A,B,C) = \bar{A}B + BC$ $\Rightarrow$ convert to canonical form.

$f(A,B,C) = \bar{A}B + BC$

$= \bar{A}B(C+\bar{C}) + BC(A+\bar{A})$

$= \bar{A}BC + \bar{A}B\bar{C} + ABC + \bar{A}BC$ $\quad$ Same order ABC

$= \bar{A}BC + \bar{A}B\bar{C} + ABC$

$= ABC + \bar{A}B\bar{C} + \bar{A}BC$ (canonical sop)

$\Rightarrow$ $f(A,B,C) = (A+\bar{B})(B+C)$

$= (A+\bar{B}+C\bar{C})(B+C+A\bar{A})$ (Apply distribution law)

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+C)(\bar{A}+B+C)$

$= \pi[0,2,3,4) = \Sigma(1,5,6,7)$

2^n – min & Max terms

2^3 8 terms

$2 \cdot N^n$

$0 - A'$, $1 - A$ (Minterms); $\Rightarrow 0 - A$, $1 - A'$ (Maxterms)

A	B	C	Minterms	Maxterms
0	0	0	$m_0 = A'B'C'$	$M_0 = A+B+C$
0	0	1	$m_1 = A'B'C$	$M_1 = A+B+C'$
0	1	0	$m_2 = A'BC'$	$M_2 = A+B'+C$
~~1~~	~~0~~	~~0~~		
~~1~~	~~0~~	~~1~~		
0	1	1	$m_3 = A'BC$	$M_3 = A+B'+C'$
1	0	0	$m_4 = AB'C'$	$M_4 = A'+B+C$
1	0	1	$m_5 = AB'C$	$M_5 - A'+B+C'$
1	1	0	$m_6 = ABC'$	$M_6 = A'+B'+C$
1	1	1	$m_7 = ABC$	$M_7 = A'+B'+C'$

$\boxed{\Sigma \Rightarrow \text{sop Minterms sum of ?}}$

$\boxed{\Pi \Rightarrow \text{pos product of Maxterms}}$

$\Rightarrow$ $f(A,B,C) = ABC + \bar{A}B\bar{C} + \bar{A}BC$
$= m_7 + m_2 + m_3$
$= \Sigma m(2,3,7)$

000 – $x'y'$
01 – $x'y$
10 – xy'
11 – xy
sop

$\Rightarrow$ $M = xy + x'z$

Express the function in sum of minterms and product of max terms.

$M = xy + x'z$
$= \cancel{m} \; xy(z+z') + x'z(y+y')$
$= \cancel{xyz + xyz' + x'zy + x}$
$= xyz + xyz' + x'yz + x'y'z$
$= m_7 + m_6 + m_3 + m_1$
$= \Sigma m(1,3,6,7)$.

$M = \Pi M(0,2,4,5)$

$\Rightarrow$ $f(A,B,C,D) = B'B + A'B + BD$

$\Rightarrow$ $f = A + B'C$

K-Map (Karnaugh Map)

1. 2 variable K-map

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	m_0	m_1
A 1	m_2	m_3

2. 3 variable K-map

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	m_0	m_1	m_3	m_2
A 1	m_4	m_5	m_7	m_6

1. Simplify using K-map

$F(x,y) = \Sigma(1,2,3)$

$F = m_1 + m_2 + m_3$

$= \bar{x}y + x\bar{y} + xy$

K-map

x \ y	$\bar{y}$ 0	y 1
$\bar{x}$ 0	0	1
x 1	1	1

$m_1 + m_3 = \bar{x}y + xy$
$= (\bar{x}+x)y$
$= y$

$m_2 + m_3$
$= x\bar{y} + xy$
$= (\bar{y}+y)x = x$

$F = x + y$

x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

$z = \Sigma(1,2,3)$. max term $\Pi(0)$

2. $F(x,y,z) = \Sigma(2,3,4,5)$

x \ yz	00	01	11	10
$\bar{x}$ 0	0	0	1	1
x 1	1	1	0	0

$\bar{x}y$

$x\bar{y}$

$F = \bar{x}y + x\bar{y}$

3. $F = \Sigma(3,4,6,7)$

x \ yz	00	01	11	10
$\bar{x}$ 0	0	0	1	0
x 1	1	0	1	1

$x\bar{z}$

yz

$F = x\bar{z} + yz$

4) $F(x,y,z) = \Sigma(0,2,4,5,6)$

$x \backslash yz$	00	01	11	10
0	1	0	0	1
1	1	1	0	1

$\bar{z}$

$x\bar{y}$

$F = x\bar{y} + \bar{z}$

5) $F(x,y,z) = 1 \Longrightarrow$

$F = \Sigma(0,1,2,3,4,5,6,7)$

6) $F = A\bar{C} + \bar{A}B + A\bar{B}C + BC$

$= A\bar{C}(B+\bar{B}) + \bar{A}B(C+\bar{C})$

$+ A\bar{B}C + BC(A+\bar{A})$

$= AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$

$+ A\bar{B}C + ABC + \bar{A}BC$

$= AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}C + ABC$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$				
A				

$A \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0		
A				

$A \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	1	1
A	1	1	1	1

B

A

$A+B$

$F = A+B$

7) $F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$

$wx \backslash yz$	00 ($\bar{y}\bar{z}$)	01 ($\bar{y}z$)	11 (yz)	10 ($y\bar{z}$)
00 ($\bar{w}\bar{x}$)	1	1	0	1
01 ($\bar{w}x$)	1	1	0	1
11 (wx)	1	1	0	1
10 ($w\bar{x}$)	1	1	0	0

$\bar{w}\bar{z}$

$x\bar{z}$

$\bar{y}$

$F = \bar{w}\bar{z} + x\bar{z} + \bar{y}$

8) $f = f(A,B,C,D)$
$= \Sigma(0,2,3,4,6,8,11,12,14)$.

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	0 (1)	1 (3)	1 (2)
$\bar{A}B$ 01	1 (4)	0 (5)	0 (7)	1 (6)
AB 11	1 (12)	0 (13)	0 (15)	1 (14)
$A\bar{B}$ 10	1 (8)	0 (9)	1 (11)	0 (10)

$\bar{A}\bar{D}$

$B\bar{D}$

$\bar{C}\bar{D}$

$\bar{B}CD$

$$F = \bar{A}\bar{D} + \bar{C}\bar{D} + \bar{B}CD + B\bar{D}$$

9) Simplify
$F(A,B,C,D) = \Pi(1,5,7,9,13,15)$ to sop form

Ans:- $= \Sigma(0,2,3,4,6,8,10,11,12,14)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	0 (1)	1 (3)	1 (2)
$\bar{A}B$ 01	1 (4)	0 (5)	0 (7)	1 (6)
AB 11	1 (12)	0 (13)	0 (15)	1 (14)
$A\bar{B}$ 10	1 (8)	0 (9)	1 (11)	1 (10)

$\bar{D}$

~~$\bar{B}CD$~~ $\bar{B}C$

~~$F = \bar{B}CD$~~

$$F = \bar{B}C + D$$

Function 'f' from k-map

1. 'f' in sop form
solve for 1's, product term

2. 'f' in pos form
solve for zeros, sum term

$\bar{F}$ from k map

1. $\bar{f}$ in sop form
solve for 0's, product term

2. $\bar{f}$ in pos form
solve for 1's, sum term

1. Implement the following functions f and its complement $\bar{f}$ using NAND Gate and NOR gate. Find SOP and POS forms of f and $\bar{f}$

i) $f(x,y,z) = \Sigma(0,6)$

NAND → SOP
NOR → POS

1) f in SOP form

$x \backslash yz$	00	01	11	10
$\bar{x}$ 0	$\boxed{1}_0$	0_1	0_3	0_2
x 1	0_4	0_5	0_7	$\boxed{1}_6$

$\bar{x}\bar{y}\bar{z}$ $xy\bar{z}$

$$F = \bar{x}\bar{y}\bar{z} + xy\bar{z}$$

2) f in POS form

$x \backslash yz$	00	01	11	10
$\bar{x}$ 0	1_0	0_1	0_3	0_2
x 1	0_4	0_5	0_7	1_6

$x+\bar{y}$ $\bar{x}+y$ $\bar{z}$

$$F = (\bar{x}+y) \cdot \bar{z}\,(x+\bar{y})$$

3) $\bar{f}$ in SOP form

$$F = \overline{\bar{x}\bar{y}\bar{z}} + \overline{xy\bar{z}}$$

$$= \cancel{x+y+z + \bar{x}+\bar{y}+z} \quad = (x+y+z)(\bar{x}+\bar{y}+z)$$

$$= x\bar{y} + z + \bar{x}y$$

3) $\bar{f}$ in SOP form

$x \backslash yz$	00	01	11	10
$\bar{x}$ 0	1_0	0_1	0_3	0_2
x 1	0_4	0_5	0_7	1_6

$\bar{x}y$ $x\bar{y}$ z

$\bar{F} = x\bar{y} + z + \bar{x}y$

$\bar{F} = (x+y+z)$
$(\bar{x}+\bar{y}+z)$

$$F = x\bar{y} + z + \bar{x}y$$

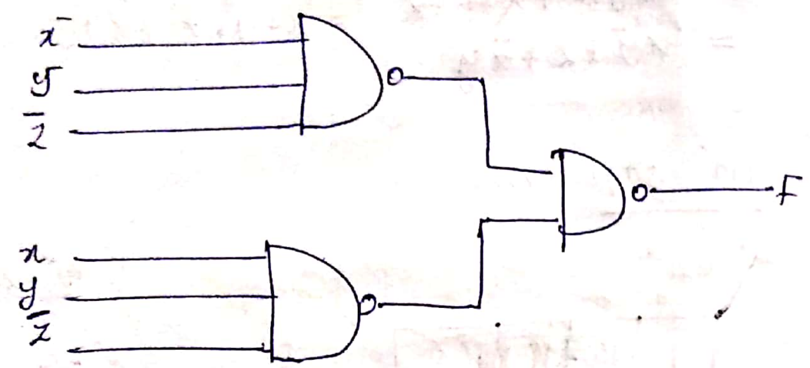
4) $\bar{f}$ in POS form

$x \backslash yz$	00	01	11	10
$\bar{x}$ 0	$\boxed{1}_0$	0_1	0_3	0_2
x 1	0_4	0_5	0_7	$\boxed{1}_6$

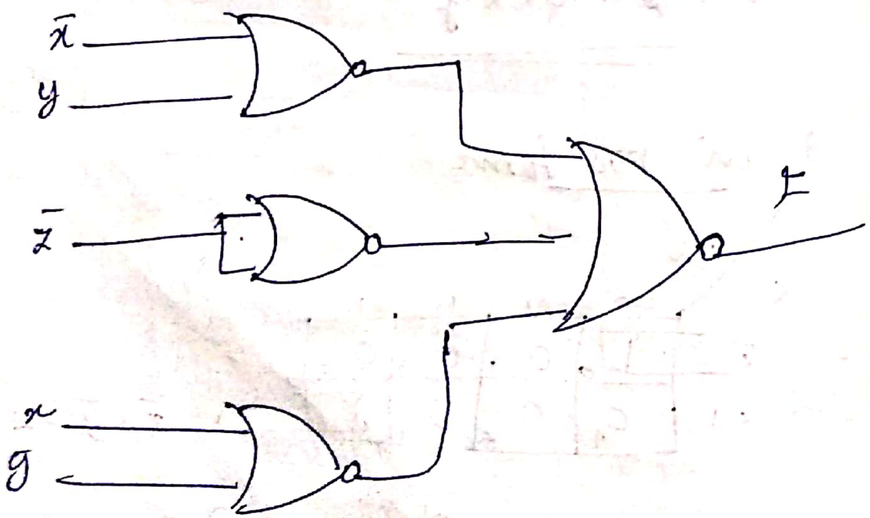
$x+y+z$ $\bar{x}+\bar{y}+z$

$$F = (x+y+z) \cdot (\bar{x}+\bar{y}+z)$$

NAND implementation of F



NOR implementation of F



⇒ ~~Simplify the following function~~

Don't Care Conditions

There are some applications where functions are not specified for a certain combinations of variables. Since these unallowed states will never require to occur in an application they can be treated as don't care term w.r.t their effect on the output i.e, for these don't care term either a '1' or a '0' may be assigned to the output. It really doesn't matter since they will never occur.

To distinguish don't care conditions from '1' and '0's a symbol 'x' is used to represent.

1. $f(w,x,y,z) = \Sigma(1,3,7,11,15)$

$d(x,y,z) = \Sigma(0,2)$

F: wx \ yz	00	01	11	10
00 ($\bar{w}\bar{x}$)	X (0)	1 (1)	1 (3)	X (2)
01	0 (4)	0 (5)	1 (7)	0 (6)
11	0 (12)	0 (13)	1 (15)	0 (14)
10	0 (8)	0 (9)	1 (11)	0 (10)

yz

$F = \bar{w}\bar{x} + yz$

2. Simplify the following function $f(A,B,C,D) = \Sigma(6,7,8,9)$ assuming that the condition having both $A=0$ and $B=0$ can never occur. Also realize the circuit, using NAND Gate.

$A=0 \quad B=0 \Rightarrow$ never occur

A	B	CD	
0	0	00	$\rightarrow m_0$
0	0	01	$\rightarrow m_1$
0	0	10	$\rightarrow m_2$
0	0	11	$\rightarrow m_3$

} don't care

F

AB \ CD	00	01	11	10
00	X (0)	X (1)	X (3)	X (2)
01	0 (4)	0 (5)	1 (7)	1 (6)
11	0 (12)	0 (13)	0 (15)	0 (14)
10	1 (8)	1 (9)	0 (11)	0 (10)

$\rightarrow \bar{A}C$

$B\bar{C}$

$Y = B\bar{C} + \bar{A}C$

3. $f = \Sigma(0,6,8,13,14)$

$d = \Sigma(2,4,10)$

WX \ yz	00	01	11	10
00	1 (0)	0 (1)	0 (3)	X (2)
01	X (4)	0 (5)	0 (7)	1 (6)
11	0 (12)	1 (13)	0 (15)	1 (14)
10	1 (8)	0 (9)	0 (11)	X (10)

$\rightarrow w\bar{z}$

$\rightarrow y\bar{z}$

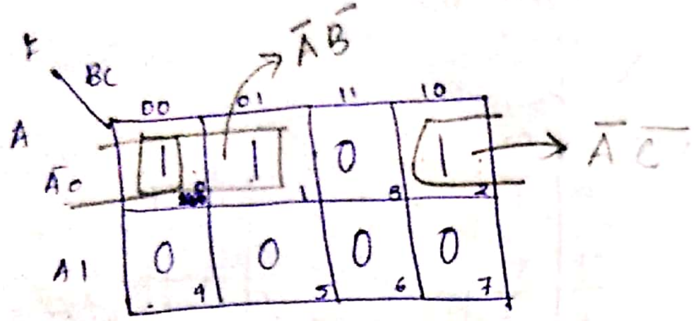
$\rightarrow w\bar{x}\bar{z}$

$wx\bar{y}z$

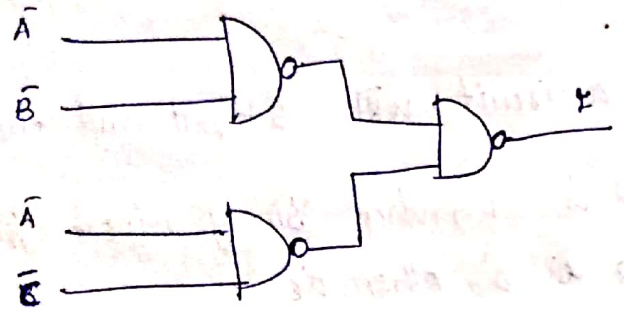
4. Design a circuit with 3 input and one output. Then o/p is 1, when binary value of i/p is < 3. The o/p is '0', otherwise.

i/p			o/p
A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$f = \Sigma(0,1,2)$



$f = \bar{A}\bar{B} + \bar{A}\bar{C}$



IMP **Quine - Mclausky Method (Tabulation Method)**

⇒ Simplify the function $f = \Sigma(0,1,6,7,8,9,13,14,15)$ using tabulation method.

$f = \Sigma\ 0,1,6,7,8,9,13,14,15$

0 - 0000
1 - 0001
6 - 0110
7 - 0111
8 - 1000
9 - 1001
13 - 1101
14 - 1110
15 - 1111

Index
0 - 0
1 - 1,8
2 - 6,9
3 - 7,14,13
4 - 15

1001 +1
1010 +1
1011 +1
1100 +1
1101 +
1110.

1110 f

Index	Minterm	Binary designation ABCD		
Index 0	0	0000✓	(0,1) 000– ; (0,8) –000	(0,1,8,9) –00– ; (0,8)(1,9) –00–
Index 1	1 8	0001✓ 1000✓	(1,9) –001 ; (8,9) 100–	
Index 2	6 9	0110✓ 1001✓	(6,7) 011– ; (6,14) –110 ; (9,13) P 1–01	(6,7)(14,15) –11– ; (6,14 7,15) –11–
Index 3	7 13 14	0111✓ 1101✓ 1110✓	(7,15) –111 ; (13,15) Q 11–1 ; (14,15) 111–	
Index 4	15	1111✓		

Prime implicants

P	1–01	$A\bar{C}D$
Q	11–1	ABD
R	–00–	$\bar{B}\bar{C}$
S	–11–	BC

Prime Implicants Chart

	0	1	6	7	8	9	13	14	15
(9,13) $A\bar{C}D$						X	X		
(13,15) ABD							X		X
*essential prime implicant (0,1,8,9) $\bar{B}\bar{C}$	X	X			X	X			
*(6,7,14,15) BC			X	X				X	X
	✓	✓	✓	✓	✓	✓		✓	✓

$BC = 1\,0$

$F = \bar{B}\bar{C} + BC + A\bar{C}D$

or

$F = \bar{B}\bar{C} + BC + ABD$

2) Simplify using tabulation method for

$F(w,x,y,z) = \Sigma(1,4,6,7,8,9,10,11,15)$

1 – 0001	Index
4 – 0100	1 – 1,4,8
6 – 0110	2 – 6,9,10
7 – 0111	3 – 7,11
8 – 1000	4 – 15
9 – 1001	
10 – 1010	
11 – 1011	
15 – 1111	

w x y z
8 4 2 1

2^3
8 0001
9 1001

difference should be power of 2

1010
1011

Always take lower & high.

	Minterms		
Index 1	1 ✓ 4 ✓ 8 ✓	(1,9) (8) P (4,6) (2) Q (8,10) (2) ✓ (8,9) (1) ✓	(8,10,9,11) (2,1) (8,9,10,11) (2,1)
Index 2	6 ✓ 9 ✓ 10 ✓	(6,7) (1) R (9,11) (2) ✓ (10,11) (1) ✓	
Index 3	7 ✓ 11 ✓	(7,15) (8) S (11,15) (4) T	
Index 4	15 ✓		

0111
1111

8 4 2 1
1000
0001
1-001

Prime Implicants

		$wxyz$		
P	- (1,9)(8)	-001	$\rightarrow$	$\bar{x}\bar{y}z$
Q	- (4,6)(2)	01-0	$\rightarrow$	$\bar{w}x\bar{z}$
R	- (6,7)(1)	011-	$\rightarrow$	$\bar{w}xy$
S	. (7,15)(8)	-111	$\rightarrow$	xyz
T	(11,15)(4)	1-11	$\rightarrow$	wyz
U	(8,9,10,11)(2,1)	10--	$\rightarrow$	$w\bar{x}$

	1	4	6	7	8	9	10	11	15
* (1,9) $\bar{x}\bar{y}z$	X					X			
* (4,6) $\bar{w}x\bar{z}$		X	X						
(6,7) $\bar{w}xy$			X	X					
* (7,15) xyz				X					X
(11,15) wyz								X	X
* 8,9,10,11 $w\bar{x}$					X	X	X	X	
	✓	✓	✓	✓	✓	✓	✓	✓	✓

1000

Essential prime Implicants

$\bar{x}\bar{y}z$

$\bar{w}x\bar{z}$

xyz

$w\bar{x}$

$f = \bar{x}\bar{y}z + \bar{w}x\bar{z} + xyz + w\bar{x}$

3) $F = \Sigma(6,7,8,9) + d(10,11,12,13,14,15)$

6 - 0110
7 - 1110
8 - 1000
9 - 1001
10 - 1010
11 - 1011
12 - 1100
13 - 1101
14 - 1110
15 - 1111

Index

1 - 8,
2 - 6, 9, 10, 12,
3 - 7, 11, 13, 14
4 - 15

1011+
1100+
1101+
1110

~~Index~~	Minterms.		
Index 1	8 ✓	(8,9)(1) ✓ (8,10)(2) ✓ (8,12)(4) ✓	(8,10,9,11)(2,1) ✓ (8,9,10,11)(1,2) ✓ (8,9,12,13)(1,4) ✓ (8,12,10,14)(4,2) ✓
Index 2	6 9 ✓ 10 ✓ 12 ✓	(6,7)(1) ✓ (6,14)(8) ✓ (9,11)(2) ✓ (9,13)(4) ✓ (10,11)(1) (10,14)(4) (12,13)(1) (12,14)(2)	(10,11,14,15)(1,4) ✓ (6,14,7,15)(8,1) P (9,13,11,15)(4,12) ✓ (9,11,13,15)(2,4) ✓ (12,13,14,15)(1,2) ✓
Index 3	7 ✓ 11 ✓ 13 ✓ 14 ✓	(7,15)(8) ✓ (11,15)(4) ✓ (13,15)(2) ✓ (14,15)(1) ✓	
Index 4	15 ✓		

~~(8,9,10,11,12,13,14,15)~~ (

(8,9,10,11,12,13,14,15) (1,2,4)

8 4 2 1
0 1 1 0
8 4 2 1
1 0 0 0

Prime implicants

	wxyz		
(6,14,7,15)(8,1)	-11-	→	xy
(8,9,10,11,12,13,14,15) (1,2,4)	1---	→	w

Prime implicants charts

	6	7	14	15
*(6,14,7,15)(8,1) xy	×	×		
(8,9,10,11,12,13,14,15) (1,2,4)			×	×
	✓	✓	✓	✓

Essential prime implicant

xy
w

$f = w + xy$

A+BC

1. Design a combinational circuit with 3 inputs and one output. The output is '1' when majority of inputs are high, otherwise the output is '0'.

input			output
A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$F = \Sigma(3,5,6,7)$

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	1

BC

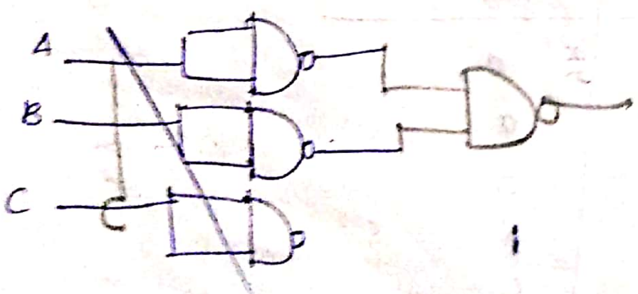
AB

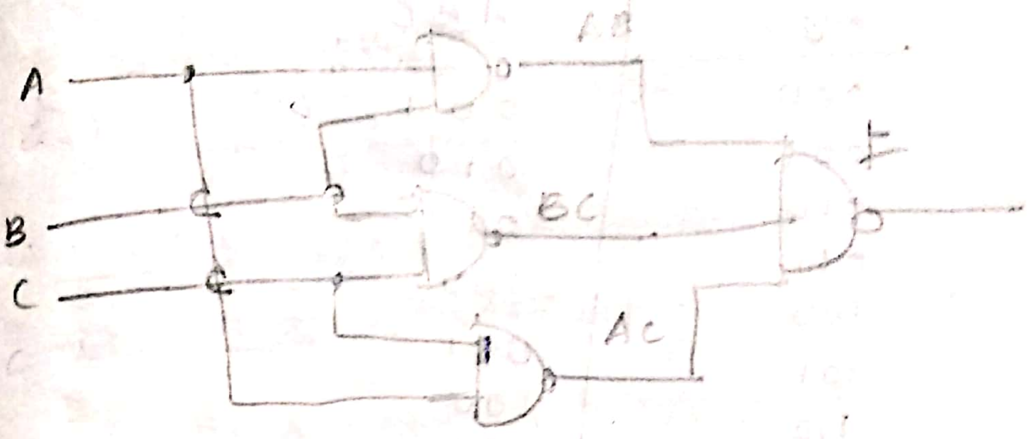
AC

~~F = A+BC~~

~~F = A$\bar{B}$C + AB + BC~~

$\therefore F = AC + AB + BC$





2. Design a circuit with 3 inputs x y z & 3 outputs ABC. when the binary input is 0,1,2 or 3, the binary output is '1' greater than the input. when the binary input is 4,5,6,or 7 the binary o/p is '1' less than the input

i/p xyz	o/p ABC
000	001
001	010
010	011
011	100
100	0[illegible]1
101	100
110	101
111	110

A

$x \backslash yz$	00	01	11	10
0	0	0	1	0
1	0	1	1	1

yz xz xy

$A = yz + xz + xy$

B

$x \backslash yz$	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$\bar{x}\bar{y}z$ $\bar{x}y\bar{z}$ $x\bar{y}\bar{z}$ xyz

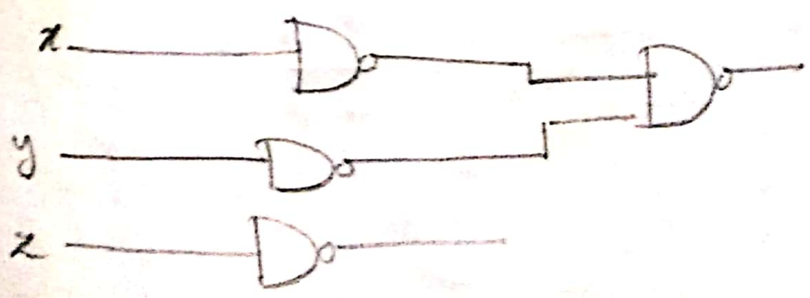
$B = \bar{x}\bar{y}z + \bar{x}y\bar{z} + xyz + x\bar{y}\bar{z} = x \oplus y \oplus z$

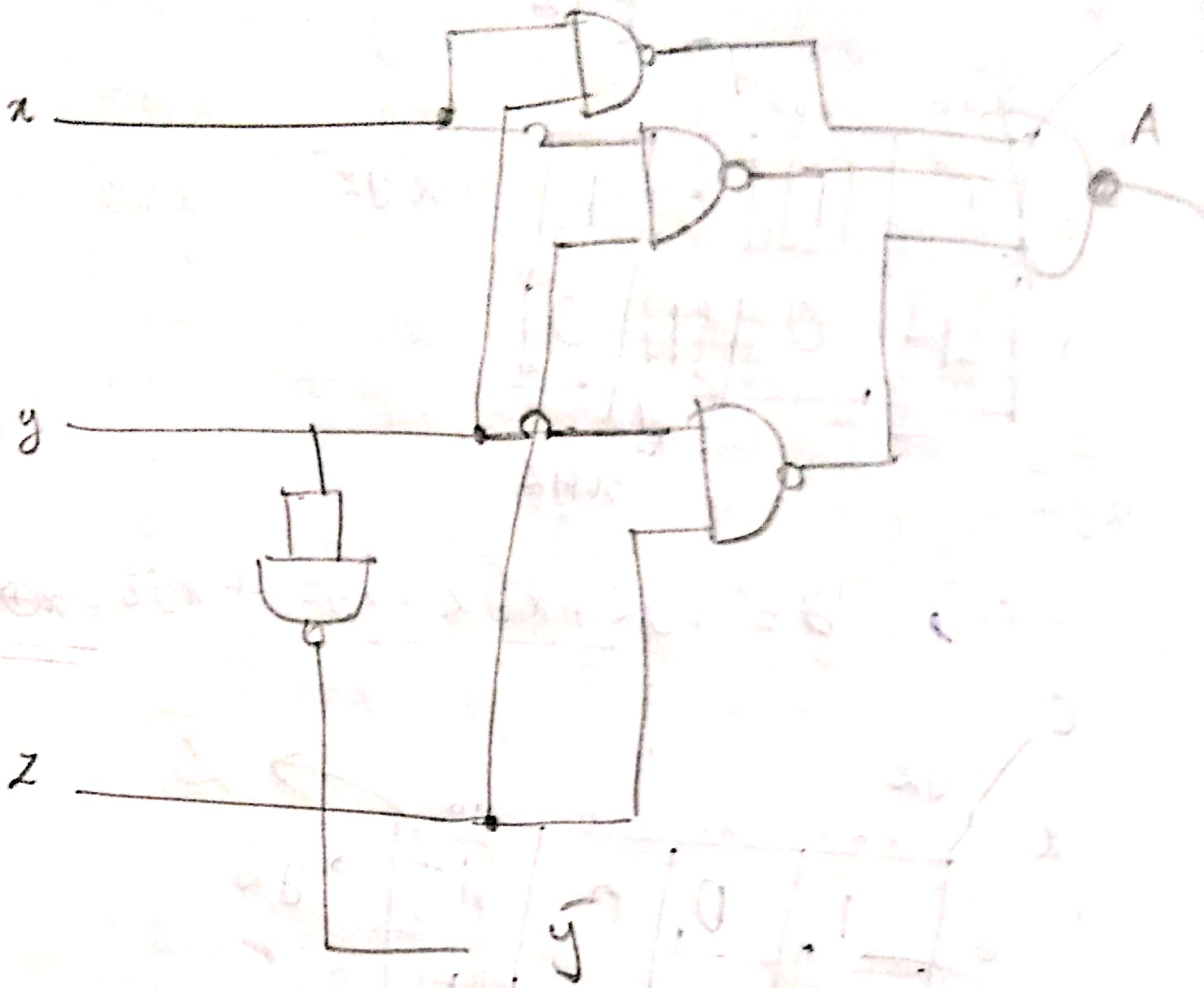
C

$x \backslash yz$	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$\bar{Z}$ $y\bar{z}$ $\bar{y}\bar{z}$

$C = y\bar{z} + \bar{y}\bar{z} = (y + \bar{y})\bar{z}$
$= \bar{z}$




x
y
z
A
ȳ