MODULE - I
NUMBER SYSTEM

Number System	Symbol	Base (Radix) r
Decimal	0-9	10
Binary	0,1	2
Octal	0-7	8
Hexadecimal	0-9, A, B, C, D, E, F	16

10^n - digits

2^n - binary digits - bits

0 - 00
1 - 01
2 - 10
3 - 11

0 - 000
1 - 001
2 - 010
3 - 011
4 - 100
5 - 101
6 - 110
7 - 111

$2^3 = 8$

$2^4 = 16$ (0-15)

Decimal to Binary

1 $(25)_{10} = (\quad)_2$

2 | 25
2 | 12 — 1
2 | 6 → 0
2 | 3 — 0
2 | 1 → 1
0 — 1

$= (011001)_2$

2. $(152)_{10} = (\quad)_2$

2 | 152
2 | 76 — 0
2 | 38 — 0
2 | 19 — 0
2 | 9 — 1
2 | 4 — 1
2 | 2 — 0
2 | 1 — 0
0 — 1

$= (10011000)_2$

3. $(0.25)_{10} = (0.01)_2$

$0.25 \times 2 = 0.50 \rightarrow 0$
$0.50 \times 2 = 1.00 \rightarrow 1$

4. $(152.25)_{10} = (10011000.01)_2$

5. $(75.35)_{10} = (\underline{\quad\quad})_2$

$0.35 \times 2 = 0.70 \rightarrow 0$
$0.70 \times 2 = 1.40 \rightarrow 1$
$0.40 \times 2 = 0.80 \rightarrow 0$
$0.80 \times 2 = 1.60 \rightarrow 1$

$(75.35)_{10} = (1001011.01)_2$

2 | 75
2 | 37 — 1
2 | 18 — 1
2 | 9 — 0
2 | 4 — 1
2 | 2 — 0
2 | 1 — 0
0 — 1

6. $(182.45)_{10} = (\underline{\quad\quad})_2$

$0.45 \times 2 = 0.90 \rightarrow 0$
$0.90 \times 2 = 1.80 \rightarrow 1$
$0.80 \times 2 = 1.60 \rightarrow 1$

$(182.45)_{10} = (10110110.011)_2$

2 | 182
2 | 91 — 0
2 | 45 — 1
2 | 22 — 1
2 | 11 — 0
2 | 5 — 1
2 | 2 — 1
2 | 1 — 0
0 — 1

Binary to Decimal

1. $(11010)_2 = (\underline{26})_{10}$

$(\overset{4}{1}\overset{3}{1}\overset{2}{0}\overset{1}{1}\overset{0}{0})_2$

$2^4 2^3 2^2 2^1 2^0$

$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 16 + 8 + 2$

$= (26)_{10}$

2. $(1010.110)_2 = (10.75)_{10}$

$2^3 2^2 2^1 2^0 \; 2^{-1} 2^{-2} 2^{-3}$

$1\times2^3+0\times2^2+1\times2^1+0\times2^0 = 8+2$

$+$

$= 10$

$1\times2^{-1}+1\times2^{-2}+0\times2^{-3}$

$= 0.5+0.25$

$= 0.75$

8 | 945
8
14
8
65

8 | 118
8
56
32
6

Decimal to Octal

1. $(7563)_{10} = (16613)_8$

$$
\begin{array}{r|l}
8 & 7563 \\
8 & 945 \; -3 \\
8 & 118 \; -1 \\
8 & 14 \; -6 \\
 & 1 \; -6
\end{array}
$$

2. $(0.52)_{10} = (0.412)_8$

$0.52\times8 = 4.16 \rightarrow 4$

$0.16\times8 = 1.28 \rightarrow 1$

~~$0.28\times8 = 0.96$~~ $\rightarrow 0$

$0.28\times8 = 2.24 \rightarrow 2$

0.52 × 8 = 4.16

0.16 × 8 = 1.28

0.16 × 8 = 1.28

Octal to Decimal

1. $(2703.51)_8 = (1475.6406)_{10}$

$8^3 8^2 8^1 8^0 \; 8^{-1} 8^{-2}$

16 | 145

$2\times8^3+7\times8^2+0\times8^1+3\times8^0+5\times8^{-1}+1\times8^{-2}$

$=1024+448+0+3+0.625+\overset{0.0156}{\cancel{0.0625}}$

$=$ ~~1475.6405~~ $= 1475.6406$

16 | 2325 = 145
16
072
64
85
80
5

Decimal to Hexadecimal

1. $(2325)_{10} = (915)_{16}$

$$
\begin{array}{r|l}
16 & 2325 \\
16 & 145 \; -5 \\
16 & \cancel{144} \; -1 \\
16 & 9 \; -1
\end{array}
$$

2. $(46)_{10} = (2E)_{16}$

16 | 46
16 | 32 −14
2

16 | 46
2 −14
2 − E

I) Decimal to Binary

$(20.31)_{10}$, $(135.02)_{10}$

II) Binary to Decimal

$(11011.001)_2$, $(1001.10)_2$

III) **Decimal to Octal**

$(5235.75)_{10}$, $(75.2)_{10}$

IV) **Octal to Decimal**

$(271.10)_8$, $(173.001)_8$

I) 1. $(20.31)_{10} = (\underline{10100.010})_2$

$0.31 \times 20 = 0.62 \rightarrow 0$

$0.62 \times 2 = 1.24 \rightarrow 1$

$0.24 \times 2 = 0.48 \rightarrow 0$

2|20
2|10 → 0
2|5 → 0
2|2 → 1
1 → 0

2. $(135.02)_{10} = (\underline{10000111.000001})_2$

$0.02 \times 2 = 0.04 \rightarrow 0$

$0.04 \times 2 = 0.08 \rightarrow 0$

$0.08 \times 2 = 0.16 \rightarrow 0$

$0.16 \times 2 = 0.32 \rightarrow 0$

$0.32 \times 2 = 0.64 \rightarrow 0$

$0.64 \times 2 = 1.28 \rightarrow 1$

2|135
2|67 → 1
2|33 → 1
2|16 → 1
2|8 → 0
2|4 → 0
2|2 → 0
1 → 0

II. 1) $(\overset{4}{1}\overset{3}{1}\overset{2}{0}\overset{1}{1}\overset{0}{1}.\overset{-1}{0}\overset{-2}{0}\overset{-3}{1})_2 = (\underline{27.125})_{10}$

$2^4\,2^3\,2^2\,2^1\,2^0 \quad 2^{-1}\,2^{-2}\,2^{-3}$

$1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 + 0\times2^{-1} + 0\times2^{-2} + 1\times2^{-3}$

$= 16 + 8 + 2 + 1 + 0.125$

$= \underline{\underline{27.125}}$

2) $(\overset{3}{1}\overset{2}{0}\overset{1}{0}\overset{0}{1}.\overset{-1}{1}\overset{-2}{0})_2 = (\underline{9.5})_{10}$

$2^3\,2^2\,2^1\,2^0\,2^{-1}\,2^{-2}$

$1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0 + 1\times2^{-1} + 0\times2^{-2}$

$= 8 + 1 + 0.5$

$= \underline{\underline{9.5}}$

654
8|5235
48
43
40
35
32
3

0.75 ×
8
0

III. 1. $(5235.75)_{10} = (\underline{12163.}\quad)_8$

$0.75 \times 8 = 6.00 \rightarrow 6$

8|5235
8|654 — 3
8|81 → 6
8|10 → 1
1 → 2

2) $(75.2)_{10} = (\underline{113.146})_8$

$0.2 \times 8 = 1.6 \rightarrow 1$

$0.6 \times 8 = 4.8 \rightarrow 4$

$0.8 \times 8 = 6.4 \rightarrow 6$

8|75
8|9 → 3
1 → 1

IV)

1. $(\overset{2}{2}\overset{1}{7}\overset{0}{1}.\overset{-1}{1}\overset{-2}{0})_8 = (\underline{185.125})_{10}$

$8^2\,8^1\,8^0\,8^{-1}\,8^{-2}$

$2\times8^2+7\times8^1+1\times8^0+1\times8^{-1}+0\times8^{-2}$

$=128+56+1+0.125$

$=\underline{\underline{185.125}}$

2. $(173.001)_8 = (\underline{120.001})_{10}$

$\cancel{(\overset{2}{1}\overset{1}{7}\overset{0}{3}.\overset{-1}{0}\overset{-2}{0}\overset{-3}{1})_8}$

$(\overset{2}{1}\overset{1}{7}\overset{0}{3}.\overset{-1}{0}\overset{-2}{0}\overset{-3}{1})_8$

$8^2\,8^1\,8^0\;\;8^{-1}8^{-2}8^{-3}$

$=1\times8^2+7\times8^1+3\times8^0+0\times8^{-1}+0\times8^{-2}+1\times8^{-3}$

$=64+56+0.001$

$=\underline{\underline{120.001}}$

Hexadecimal to Decimal

1. $(\overset{2}{2}\overset{1}{E}\overset{0}{F}.\overset{-1}{0}\overset{-2}{1})_{16} = (\underline{\qquad})_{10}$

A - 10
B - 11
C - 12
D - 13
E - 14
F - 15

$2\times16^2+14\times16^1+\overset{15}{\cancel{13}}\times16^0+0\times16^{-1}+1\times16^{-2}$

$=512+224+\overset{15}{\cancel{15}}+0.003$

$=\cancel{751.003}$ $\cancel{736.003}$

$=\underline{\underline{751.003}}$

Binary to Octal

1. $(1110111)_2 = (\underline{367})_8$

$(\underbrace{011}\,\underbrace{110}\,\underbrace{111})$

$\downarrow\;\downarrow\;\downarrow$

$3\;\;6\;\;7$

3bit binary

2. $(11001.01011)_2 = (\underline{\qquad})_8$

$(\underbrace{011}_{3}\,\underbrace{001}_{1}.\underbrace{010}_{2}\,\underbrace{110}_{6}) = (31.26)_8$

Octal to Binary

$(2571.2)_8 = (\_\_\_\_\_\_)_2$

$$\left(\underset{\downarrow\ \downarrow\ \downarrow\ \downarrow}{2571.2}\right)_8 = (010101111001.010)_2$$

010101111001.010

Binary to Hexadecimal

1. $(010111111011.011111)_2$

$= (\underline{\ 2FB.7C\ })_{16}$

$(\underset{1}{001}\,\underset{3}{011}\,\underset{7}{111}\,\underset{3}{011}\,.\,\underset{3}{011}\,\underset{7}{111})$

$(\underset{2}{0010}\,\underset{F}{1111}\,\underset{B}{1011}\,.\,\underset{7}{0111}\,\underset{C}{1100})_2$

(2 F B ~~B7C~~) 7 C

4 bit binary

0000
0001
0010
0100
1000

0000
0001
0010
0011
0100

Hexadecimal to Octal
Hexadecimal → Binary → Octal.

Hexadecimal to Binary

$(8E47.0AB)_{16} = (\_\_\_\_\_\_)_2$

1000 1110 0100 0111 . 0000 1010 1011

$= (1000111001000111.000010101011)$

0000 – 1

⇒ convert decimal number ~~805~~ 8567 to base 5 system

$(8567)_{10} = (233232)_5$

5 | 8567
5 | 1713 → 2
5 | 342 → 3
5 | 68 → 2
5 | 13 → 3
2 → 3

⇒ $(\overset{3^3\,3^2\,3^1\,3^0}{\underset{3\ 2\ 1\ 0}{1212}})_3 = (\_\_\_\_)_{10}$

$1\times 3^3 + 2\times 3^2 + 1\times 3^1 + 2\times 3^0$

$= 27 + 18 + 3 + 2$

$=$ ~~26~~ 50

Binary Arithmetic

Binary Addition

	Sum	Carry
0+0	0	0
0+1	1	0
1+0	1	0
1+1	0	1

1.
0 0
011 + → 3
001 → 1 → 4 100
100

2.
① ① ①
1111 → 15
1101 → 13 = 28
11100

3.
1 1 1 1
11011
1101
101000

①
① = 2
10
3 - 11

10
11
11

4)
1 1
10111.101 +
10011.01
101010.111

5)
⑩
11
11 +
01
01
1000

	Sub	Borrow
0-0	0	0
0-1	1	1
1-0	1	0
1-1	0	0

1)
1101 −
100
1001

0 →	0000
1 →	0001
2 →	0010
3 →	0011
4 →	0100
5 →	0101
6 →	0110
7 →	0111
8 →	1000
9 →	1001
A →	1010
B →	1011
C →	1100
D →	1101
E →	1110
F →	1111

10
11
100

0011 +
1
0100

2) 110 −
1
101 = 5

6 − 1 = 5

2 − 1 = 1

110 −
1
101

3) 100 −
1
011

100 −
1
011

4) 101 −
011
010

5) 1000 −
10
0110

1010.010 −
111.111
0010.011

6) 1010.010 −
111.111
0010.011

1010.010 −
111.111
0010.011

7) 10110 − 10.11

8) 1100.10 − 111.01

9) 10001.01 − 1111.11

7) 10110.00 −
10.11
10011.01

8) 1100.10 −
111.01
101.01

9) 10001.01 −
1111.11
00001.10

Binary Multiplication

1. 101 ×
11
1111

5 × 3 = 15

101 ×
11
101 +
101
1111

2. 1011.101 ×
10.01
1011101
0000000 +
0000000
1011101
11010.00101

10 +
1
11

Binary Division

1.
```
        000111
110 | 101010
      0
      10
      0
      101
      0
      1010
       110
      1001
       110
      00110
        110
      00000
```

2)
```
        111.1
110 | 101101
      110
      1010
       110
      1001
       110
      00110
        110
      00000

000111.1
```

3)
```
           1
101 | 110101.11
       101
```

3)
```
          1
101 | 110101.11
        101
      110000.11
```

3)
```
        101
101 | 110101.11
      101
      00110
        101
      00011
```

3)
```
        1010.11
101 | 110101.11
      101
      00110
        101
       00111
         101
         0101
          101
          000
```

Octal Addition

steps

1. Similar to decimal addition
2. Add the digits in each columns, in decimal systems and convert the sum into octal
3. Record the octal sum term in that column and move the carry to the next column.

1) Add $(523)_8$ and $(216)_8$

```
  1
 523 +
 216
(741)8
```

$3+6=(9)_{10}$
$=(11)_8$

2) $(175.6)_8$ and $(47.7)_8$

```
 1 1 1
 175.6 +
  47.7
2 45.5

(245.5)8
```

$7+6=(13)_{10}$
$=(15)_8$

~~$(7+4+1)=(12)_{10}$~~
~~$=(13)_8$~~
$(7+4+1)=(12)_{10}$
$=(14)_8$

Decimal	Octal
0	0
⋮	⋮
7	7
8	10
9	11
10	12
11	13
12	14
13	15
14	16
15	17
16	20
17	21
18	22

3)
  1 1 1 1
  3 2 7 . 5 4 +
  6 6 5 . 3 7
  1 2 1 5 . 1 3

$(1215.13)_8$

$7+4=(11)_{10}$
$=(13)_8$

$5+3+1=(9)_{10}$
$=(11)_8$

$7+5+1=(13)_{10}$
$=(15)_8$

$6+3+1=(10)_{10}$
$=(12)_8$

4) $(1247)_8$ & $(2053)_8$

   1 1
  1 2 4 7 +
  2 0 5 3
  3 3 2 2

$(3322)_8$

5) $(273.56)_8$ & (425.17)

  1 1   1
  2 7 3 . 5 6 +
  4 2 5 . 1 7
  7 2 0 . 7 5

$7+6=(13)_{10}$
$=(15)_8$

$5+3=(8)_{10}$
$=(10)_8$

Octal subtraction

1 borrow
$\xrightarrow{}$ 10
In octal
10 = 8

1. $(273)_8 - (15)_8$

  6 $\cancel{7}$ 8
  2 7 3 −
    1 5
  2 5 6

$(256)_8$

$(8+3)-5 = 11-5$
$=(6)_{10}$
$=(6)_8$

2.
  7 10 8
  2 0 . 1 4 −
  1 6 . 4 7
    1 . 4 5

$(8+4)-7 = 12-7$
$=(5)_{10}$
$=(5)_8$

$(20.14)_8 - (16.47) = (1.45)_8$

  8
  20.14 −
  16.47

3.
  7 7   8
  2 0 0 . 2 −
    1 2 . 7
  1 6 5 . 3

$(8+2)-7 = 10-7$
$=(3)_{10}$
$=(3)_8$

$(200.2)_8 - (12.7)_8 = 165.3$

4.
  2 7 7 5 7 8
  3 0 0 6 . 0 5 −
  2 6 5 7 . 1 6
  0 1 2 6 . 6 7

$(8+5)-6 = (7)_{10}$
$=(7)_8$

$(8+5)-7 = 13-7$
$=(6)_{10}=(6)_8$

5. $(175.6)_8 - (47.7)_8 = (125.7)_8$

```
 5 8  8
 175.6 -
  47.7
 ------
 125.7
```

$8+6-7 = 14-7$
$= (7)_{10}$
$= (7)_8$

$8+4-7 = 12-7$
$= (5)_{10} ; (5)_8$

Hexadecimal Addition

1)
```
 1
 A C 6 +
 B 5 9
 -------
 1 6 1 F
```

$6+9 = (15)_{10}$
$= (F)_{16}$

$12+5 = (17)_{10}$
$= 11$

$A+B+1 = 10+11+1$
$= (22)_{10}$
$= (16)_{16}$

$(AC6)_{16} + (B59)_{16}$

$= (161F)_{16}$

Decimal	Hexad
0	0
⋮	⋮
9	9
10	A
11	B
12	C
13	D
14	E
15	F
16	10
17	11
18	12
19	13
20	14
21	15
22	16
23	17
24	18

2)
```
 1 1 1 1   1
 2 A 7 C . 3 0 D +
   8 D 9 . E 8 B
 -----------------
 (3 3 5 6 . 1 9 8)16
```

$D+B = 13+11$
$= (24)_{10} = (18)_{16}$

$E+3 = 14+3$
$= 17$

$C+9+1 = (22)_{10} = (16)_{16}$

$7+1+D = 13+8 = (21)_{10} = (15)_{16}$

$(3356.198)_{16}$

$A + 8 + 1 = 10 + 9$
$= (19)_{10}$
$= (13)_{16}$

Hexadecimal Subtraction

1)
```
   9 16
 F A 7 -
 B 9 E
 -------
 (4 0 9)16
```

$16+7-E$
$= 23-14$
$= (9)_{10} = (9)_{16}$

$15 - 11 = 4$

2) Subtract $(7EB6.3B)_{16}$ from $(B08E.A1)_{16}$

```
 A 15 16   9 16
 B 0  8 E . A 1 -
 7 8  D 6 . 3 B
 -----------------
 (3 7 B 8 . 6 6)16
```

$(16+1) - B$
$= 17 - 11$
$= (6)_{10}$
$= (6)_{16}$

$E - 6 = 14 - 6$
$= (8)_{10} = (8)_{16}$

3) $(CDF7.52)_{16} - (AB5.8)_{16}$

A) $(67F2.6E)_{16} - (4A0E)_{16}$

$16+8-$ 13

$= 24 - 13$

$= (11)_{10}$

$=$ ~~$(D)_{16}$~~ B

```
      6  16
 C D F 7 . 5 2 -
     A B 5 . 8
 ---------------
 (3 4 1 . B 2
```

$16+5-8$
$= 21-8$
$= (13)_{10} = (D)_{16}$

$13 - 10$
$= 3$

$F - B = 15 - 11$
$= (4)_{10} = (4)_{16}$

5) 67F2.6E −
4A0E.00

1DE4.6E

16+2 −E
= 18 − 14
= 4

16+7 −A
= 23−A
= $(13)_{10}$ = D

Complements

1. r's complement
2. (r−1)'s complement

$\boxed{\text{r's complement of a number} = r^n - N}$

r − base
n − no: of digits in the number
N − No

Binary − 1's & 2's
Decimal − 9's & 10's
Octal − 7's & 8's
Hexadecimal − 15's & 16

1) $(235)_{10}$

10's complement $(235)_{10} = 10^3 - 235$
$= 1000 - 235$
$= (765)_{10}$

2) 2's complement of $(100)_2 = 2^3 - 100$
$= (8)_{10} - (100)_2$
$= 1000 - 100$
$= 100$

1000 −
100
100

$\boxed{(r-1)\text{'s complement} = r^n - N - 1}$

$\boxed{(r-1)\text{'s complement} + 1 = r\text{'s complement}}$

1. Find 9's complement and 10's complement of following decimal numbers.
1. 4069
2. 745
3. 1056.074

1. 9's complement of 4069 = 9999 −
4069
5930

10's complement of 4069 = 9's complement + 1
= 5930 +
1
5931

2. 9's complement of 745 = 999 −
745
254

10's complement of 745 = 254 +
1
255

3. 9's compleent of 1056.074 = 9999.999 −
1056.074
8943.925

10's complement of 1056.074 = 8943.925 +
1
8943.926

2) Find 1's complement and 2's complement of the following binary numbers.

1) 1100 2) 10101 3) 1010.01

i) 1's complement of 1100 = 0011
2's complement of 1100 = 0011 +
1
0100

ii) 1's complement of 10101 = 01010
2's complement of 10101 = 01010 +
1
01011

11111 −
10101
01010

iii) 1's complement of 1010.01 = 0101.10
2's complement of 1010.01 = 0101.10 +
1
0101.11

2's compl
-(17)$_{10}$
↓
convert to binary

Signed Numbers

There are 3 ways to represent signed numbers.

1) Sign Magnitude form
2) 1's complement form
3) 2's complement form

Scanned by CamScanner

Sign Magnitude form

1) +10 -10

01010 11010

↓ sign bit, magnitude ↓ sign bit, magnitude

most significant bit

101 . . . 1
MSB

sign LSB bit

0 +ve
1 -ve

2) +41 -41

~~011001~~ ~~111001~~

0101001 1101001

$(41)_{10}$

2 | 41
2 | 20 → 1
2 | 10 → 0
2 | 5 → 0
2 | 2 → 1
1 0

1's complement form

1) +10 -10

$01010 \xrightarrow{\text{1's complement}} 10101$

2) +41 -41

0101001 1010110

3) 8 bit representation :

+10 $\overline{10}$

$00001010 \xrightarrow{\text{1's complement}} 11110101$

2's complement form

1. +10 -10

$01010 \xrightarrow{\text{1's complement}} 10101 +$

1

10110 ⇒ 2's complement

2. +41 -41

0101001 1010110 +

1

1010111 ⇒ 2's complement

$(51)_{10}$

2 | 51
2 | 25 → 1
2 | 12 → 1
2 | 6 → 0
2 | 3 → 0
1 → 1

+51 -51

0110011 1110011
~~0001100~~

$0110011 \xrightarrow{\text{1's complement}} 1001100 +$

1

1001101 ⇒ 2's complement

	Sign Magnitude form	1's complement	2's complement
+51	011011	011011	011011
-51	1110011 ~~0001100~~	1001100 ~~100100~~	1001101

⇒ Represent the following Numbers in sign magnitude, 1's compliment and 2's complement form

1. +39 & -39
2. -75 & +75
3. +45.75 & -45.75

⇒ Each of the following number is a ~~number~~ signed binary number. Determine the decimal value in each case, if they are in sign magnitude form, 2's compliment form and 1's complement form.

a) 01101 c) 10111
b) 010111 d) 1101010

a) 01101

1101

$1\times2^3+1\times2^2+0\times2^1+1\times2^0$

$= 8+4+1$

$= +13$

b) 010111

10111

$1\times2^4+0\times2^3+1\times2^2+1\times2^1+1\times2^0 =$

$= 16+4+2+1$

$= 23$//

	Sign Magnitude	1's compliment	2's complement
01101	+13	+13	+13
010111	+23	+23	+23
-ve ← 10111	-7	-8	-9
1101010	-42	-21 ~~-8~~	-22 ~~-8~~

1101010 ~~01011~~0

0010101 010101

$0\times2^5+1\times2^4+0\times2^3+1\times2^2+0\times2^1+1\times2^0$

$= 16+2+1 = 19$

010101

$0\times2^5+1\times2^4+0\times2^3+1\times2$

-7 10111 ↓ 1's compl

01000 + 1

01001 = 2's complement

10111 $\xrightarrow{\text{1's comp}}$ 01000 + 1

01001

1000

$= 1\times2^3$

$= 8$//

1001

$1\times2^3+1\times2^0$

$= 8+1 = 9$//

1101010 ↓ 1's com

0010101 + 1

0010110 2's com

010110

$0\times2^5+1\times2^4+0\times2^3+1\times2^2+1\times2^1+0\times2^0$

$= 16+2 = 18$

1101010

$1\times2^5+0\times2^4+1\times2^3+0\times2^2+1\times2^1$

$= 32+16+8+2$

$= 42$

⇒ Represent -99 and -77.25 in 8 bit 1's complement form

$1\times2^4+1\times2^2+1\times2^0$

$= 16+4+1 = -21$

1101010 $\xrightarrow{\text{1's complement}}$ 0010101 + 1

0010110 0010110

$1\times2^4+1\times2^2+1\times2^1$

$= 16+4+2$

$= 22$//

Range of values (if n bit for representation)

	Min	Max
Unsigned	0	$2^n - 1$
Signed	$-(2^{n-1}-1)$	$+(2^{n-1}-1)$
1's compliment	$-(2^{n-1}-1)$	$+(2^{n-1}-1)$
2's compliment	$-(2^{n-1})$	$+(2^{n-1}-1)$

Signed
000 +0
100 -ve
1's compl.
000
111 2's
1000 → overfl

● 1's complement Arithmetic (subtraction Using 1's compliment)

Step 1
Let be subtract. $M - S = M + (-S)$

Step-2
Take 1's complement of S

Step-3
Add M and 1's complement of S (-s)

Step. 4
If there is a carries, bring the carry around and add it to the LSB. This is called end around carry

Step 5
Look at the sign bit (MSB). If this is a zero, the result is +ve and is in the original binary format. If the MSB is a '1' (whether there is a carry or no carry at all). The result is -ve and is in its 1's complement form. Take its 1's complement to get the magnitude in original binary.

1. +6 → 00000110
   +13 → 00001101
   00010011 ⇒ 19

2. -6 +13

$$\begin{array}{rl} +6 = & 00000110 \\ -6 = & 11111001 \quad \text{1's complement} \\ +13 = & 00001101 \\ \hline \text{end carry} \leftarrow & (1)00000110 \\ & \quad\quad\quad\ 1 + \\ \hline & 00000111 \Rightarrow +7 \end{array}$$

MSB = 0 ∴ +ve (+7)

3. 6 -13

6 + -13

+6 = 00000110

+13 = 00001101 2's complement

-13 = 11110010 +

+6 = 00000110

11111000

MSB = 1 ∴ -ve

So) −(1's complement of result)

ans = −(00000111)

= −7

4. -6 -13

-6 + -13

+6 = 00000110

-6 = 11111001 2's complement

+13 = 00001101 2's complement

-13 = 11110010

-6 + -13 ⇒ 11111001 +

11110010

①11101011

MSB = 1 ∴ -ve

ans = −(11101011 +
1
11101100)

-19

Ans = −(00010011)

= −19

5. Subtract 14 from 25 using 8 bit 1's complement arithmetic

6. Add -25 and +14

7. Add -25 and -14

8. Add +25 and -25

9. -89.75 and +43.25

⇒ Represent in i) Sign magnitude ii) 1's complement iii) 2's compliment

0100111 $\xrightarrow{\text{1's comp.}}$ 1011000 +
1
1011001

2 | 39
2 | 19 → 1
2 | 9 → 1
2 | 4 → 1
2 | 2 → 0
1 → 0

1) +39 -39

	signed	1's comp	2's compliment
+39	0100111	0100111	0100111
-39	1100111	1011000	1011001

2) +75 and -75

	signed	1's	2's
+75	01001011	01001011	01001011
-75	11001011	10110100	10110101

2 | 75
2 | 37 → 1
2 | 18 → 1
2 | 9 → 0
2 | 4 → 1
2 | 2 → 0
1 → 0

01001011 ↓ 1's com

10110100 +
1

10110101

3) +45.75 and -45.75

$0101101.11 \xrightarrow{1's\ compl} 1010010.00 +$
1

1010010.01

2 | 45
2 | 22 → 1
2 | 11 → 0
2 | 5 → 1
2 | 2 → 1
1 → 0

0.75×2 = 1.5 → 1

0.5×2 = 1.0 → 1

	signed	1's	2's
+45.75	0101101.11	0101101.11	0101101.11
-45.75	1101101.11	1010010.00	1010010.01

⇒ Represent -99 & -77.25 in 8 bit 1's complementary form.

1. -99

11100011

+99

$01100011 \xrightarrow{1's\ complement}$ -99 10011100

2 | 99
2 | 49 → 1
2 | 24 → 1
2 | 12 → 0
2 | 6 → 0
2 | 3 → 0
1 → 1

2. -77.25

11001101.01

+77.25

$01001101.01 \xrightarrow{1's\ compl}$ -77.25 10110010.10

2 | 77
2 | 38 → 1
2 | 19 → 0
2 | 9 → 1
2 | 4 → 1
2 | 2 → 0
1 → 0

0.25×2 = 0.5 → 0

0.5×2 = 1.0 → 1

⇒ 1. +25 = 010001

+14 = ~~01010~~ 01110

8bit binary of +25 = 00010001

8bit binary of +14 = 00001110 2 1's compl

2 | 25
2 | 12 → 1
2 | 6 → 0
2 | 3 → 0
1 → 0

2 | 14
2 | 7 → 0
2 | 3 → 1
1 → 1

25+-14 -14 = 11110001 +

00010001 ⇒ MSB = 1

①00000010

+1

00000011 ⇒ +11 MSB=0 +ve

2) -25 +14

8 bit binary of +25 = 00010001 2's comp

" " " -25 = 11101110

8 bit bin of +14 = 00001110

```
           000
-25+14 ⇒  11101110 +
          00001110
          --------
          11111100   ⇒ MSB=1 ∴
```

Answer is -(00000011)

= -11

3) -25 + -14

8 bit binary of +25 = 00010001 2's compl

8 bit " -25 = 11101110

8 bit " " of +14 = 00001110 2's com

" " , -14 = 11110001

```
-25+-14 ⇒  ~~11~~10110 +
           ~~11~~1001
           --------
          ①11011111
                  1+
           --------
           111000 0   ⇒ MSB=1 ∴ -ve
```

~~Answer = -(0001111)~~

```
           1 ①
-25+ -14 ⇒ 11101110 +
           11110001
           --------
          ①11011111 +
                  1
           --------
           11100000   MSB=1 ∴ -ve
```

Answer = -(0011111)

=

4) 8 bit binary of +25 = 00010001

8 bit " " -25 = 11101110

```
-25+25      11111111   ⇒ MSB=1
                         ∴ -ve
```

Answer = -(00000000)

= 0

5) -89.75 + +43.25

1011001.11

0.75×2 = 1.5→1

0.5×2 = 1 →1

```
2 | 89
2 | 44 → 1
2 | 22 → 0
2 | 11 → 0
2 | 5  → 1
2 | 2  → 1
    1  → 0
```

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43.25

= 101011.01

$2 \,|\, 43$
$2 \,|\, 21 \to 1$
$2 \,|\, 10 \to 1$
$2 \,|\, 5 \to 0$
$2 \,|\, 2 \to 1$
$1 \to 0$

$0.25 \times 2 = 0.5 \to 0$
$0.5 \times 2 = 1 \to 1$

-89.75 + 43.25

= 11011001.11 +
00101011.01

①0 0000101.00 +
1

00000101.01

MSB = 1

MSB = 0 ∴ +ve

5

6) $(11010)_2 - (1101)_2$ using 1's compliment

11010
4 3 2 1 0

$1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$

= 16 + 8 + 2

= 26

-26

1101
3 2 1 0

$1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

= 8 + 4 + 1

= 5 + 8 = 13

26 - 13

---

b) +1010 $\xrightarrow{\text{1's compliment}}$ 11010

~~-10 + -5~~

11010 $\xrightarrow{\text{1's compliment}}$ 00101

1101 $\xrightarrow{\text{1's complem}}$ 0010

+10 + 15 = 00000101
00000010 +
00000111

$(11010)_2 + (-(1101)_2)$

= 00011010 +
00000010
11100

5 bit of 1101 = 01101 } 1's complent

= 10010

$(11010)_2 + (-(10010)_2)$

= 11010 +
10010
①01100 +
1
01101 +13

2's compliment arithmetic

1. -6 +13

+13 = 00001101

+6 = 00000110 } 1's complemult

~~11111001~~ + } 2's complemut

~~1~~

~~1111010~~

+6 = 00000110 } 1's complemt

-6 = 11111001

1 + } 2's complent

11111010

-6 +13 = 11111010 +

00001101

①0 0000111

end carry discard

∴ -6+13 = 00000111 $(+7)_{10}$

2) -13 +6

+13 = 00001101 } 1's complement

-13 = 11110010

1 + } 2's complent

11110011

+6 = 00000110

-13+6 = 11110011 +

00000110

11111001

MSB = 1 ∴ -ve

∴ Take (2's complemt)

1's complement 11111001 = 00000110

(2's complemnt) = 00000110 +

1

-(00000111) ~~MSB = 0~~

= -7

3) -13 -6

-13 + -6

+13 = 00001101 } 1's complement

-13 = 11110010 + } 2's complemnt

1

11110011

+6 = 00000110 } 1's complement

-6 = 11111001 + } 2's complement

1

11111010

-6 + -13 = 11111010 +

11110011

①11101101

discard end carry

since MSB = 1 ∴ (-ve)

∴ Ans = -ve (2's complement)

1's comple of 11101101 = 00010010 + } 2's comp

1

00010011 = $(-19)_{10}$

4) $(11010)_2 - (1101)_2$ using 2's complement method

$(11010)_2 + (-(1101)_2)$

5 bit of 1101 = 01101 } 1's compl

= 10010 } 2's compln

= 10010 +

1

10011

2|25
2|24-1
12-1

$(11010)_2 + (-10011)$

= 11010 +

10011

①01101 = 01101 $(+13)_{10}$

discard end carry

BCD-code: Binary Coded Decimal

BCD is a way to express each of the decimal digit with a 4 bit (0-9) binary code. There are only 10 code groups in a BCD system. So that, it is very easy to convert decimal and BCD representation.

Weighted code

4 bit
8 4 2 1
5 bit
16 8 4 2 1
6 bit
32 16 8 4 2 1

BCD means that each decimal 0-9 is represented by its straight binary in 4 bit. The 6 code combinations 1010, 1011, 1100, 1101, 1110, 1111 (10-15) are invalid in BCD. BCD is also called 8421 code and also it is a weighted code.

⇒ convert each of the following number to BCD

1) 35　　　3) 125
2) 2469　　4) 2461.35

1) 35 → $(00110101)_{BCD}$

2) 2469 → $(0010010001101001)_{BCD}$

3) 125 → $(000100100101)_{BCD}$

4) 2461.35 → $(0010010001100001 \cdot 00110101)_{BCD}$

BCD Addition

⇒ Add the following 8421 BCD numbers.

1) 23 + 15 →

①①①
00100011 +
00010101
00111000

38.

38

2) 9+4 ⇒ 1001
0100
1101 → Invalid code
→ should add 6

∴ 1101 +
0110
00010011 ⇒ $(13)_{10}$

3) 67 + 53 ⇒

1 111
01100111 +
01010011
10111010 → Invalid code
→ should add 6

∴ ~~10111010 +~~
~~01000100~~
~~11111110~~

∴
①①①①①①
10111010 +
01100110
000100100000

120 ⇒ $(120)_{10}$

4) 275 + 496 ⇒ 0010 0111 0101 +
0100 1001 0100
~~0111 0000 1010~~

carry from 4 bit ⇒ add 6

275 + 496 ⇒ 0010 0111 0101 +
0100 1001 0110
0111 0000 1011 invalid ⇒ should add 6

∴ 0111 0000 1011 +
0000 0110 0110
0111 0111 0001 ⇒ $(771)_{10}$
7 7 1

5) 679.6 + 536.8

0110 0111 1001 . 0110 +
0101 0011 0110 . 1000
1 0111 0101 1111 . 1110 ⇒ invalid ⇒ should add 6

1 0111 0101 1111 . 1110
0110 0110 0110 . 0110
0001 0010 0001 0110 . 0100
1 2 1 6 . 4

1216.4

BCD subtraction

1. 38 - 15 ⇒ 0011 1000 −
0001 0101
0001

0011 1000 −
0001 0101
0010 0011
2 3

38 − 15 = $(23)_{10}$

11010 −
00101
10101

2. 920 − 265 ⇒ 1001 0010 0000 −
0010 0110 0101
0110 1011 1011 → invalid should subtract 6
~~6~~

0110 1011 1011 −
0000 0110 0110
0100 0101 0101
A~~6~~ 5 5

0111 0100 0000 −
0010 011

0111 0100 0000 −
0010 0110 0101
0100 1011 1011

BCD subtraction using's 9's complement & 10's complement method

1. Perform the following decimal operations using 9's complement method

1. 38-15 2) 75-23

3. 206.7 - 147.8

1. 38-15

9's complement of -15 = $\begin{array}{r} 9\cancel{9} - \\ \cancel{75} \\ \hline \cancel{4} \end{array}$ $\begin{array}{r} 99 - \\ 15 \\ \hline 84 \end{array}$

∴ 38-15 ⇒ $\begin{array}{r} \overset{1}{3}8 + \\ 84 \\ \hline ①22 + \\ 1 \\ \hline 23 \end{array}$

9's comp -23 = 23

If carry no is +ve & add 1

2. 75-23

9's complement of -23 = $\begin{array}{r} 99 - \\ 23 \\ \hline 76 \end{array}$

∴ $\begin{array}{r} \overset{1}{7}5 + \\ 76 \\ \hline ①51 \end{array}$ ⇒ $\begin{array}{r} 51 + \\ 1 \\ \hline 52 \end{array}$

3. 206.7 - 147.8

9's complement of -147.8 ⇒ $\begin{array}{r} 999.9 - \\ 147.8 \\ \hline 852.1 \end{array}$

206.7 - 147.8 ⇒ $\begin{array}{r} 206.7 + \\ 852.1 \\ \hline ①058.8 + \\ 1 \\ \hline 058.9 \end{array}$ ⇒ $(58.9)_{BCD}$

4. 679.6 - 885.9

9's complement of -885.9 = $\begin{array}{r} 999.9 - \\ 885.9 \\ \hline 114.0 \end{array}$

679.6 - 885.9 ⇒ $\begin{array}{r} 6\overset{1}{7}9.6 + \\ 114.0 \\ \hline 793.6 \end{array}$ No carry

∴ answer is -ve

∴ Take 9's compl & answer

~~999.9~~ -

9's complement of 793.6 = $\begin{array}{r} 999.9 - \\ 793.6 \\ \hline 206.3 \end{array}$

Answer = -(206.3)

2) Perform the following decimal operations in 8421 BCD code using 9's complement method.

~~38-15~~

~~85-24~~

1. 85-24

2. 305.5-168.8

3. 679.6-885.9

1. 85 -24

9's complement of -24 = 99 - 24 = 75

85 → 10000101 (BCD of 85)

75 → 01110101 + (BCD of 75)

11111010 → Invalid should add 6

11111010 +

01100110

00010110000

~~160~~

15 - 24 = 61

0110 0011

01100001

61

2. 305.5 - 168.8

9's complement of -168.8 = 999.9 - 168.8 = 831.1

BCD of 305.5 = 001100000101.0101 +

BCD of 831.1 = 100000110001.0001

1011 0011 0110 . 0110 (Invalid should add 6)

3 6 6

136.7

∴ 101100110110.0110 +

0110

00 ① 0001 0011 0110 . 0110 +

①

0001 0011 0110 . 0111

136.7

3. 679.6 - 885.9

9's complement of -885.9 = 999.9 - 885.9 = 114.0

885.9 - 679.6 = 206.3

BCD of 679.6 = 011001111001.0110 +

BCD of 114 = 000100010100

0111 1000 1101 . 0110

6 Invalid add 6

0111 1000 1101 . 0110 +

0000 0000 0110 . 0000

0111 1001 0011 . 0110 = 793.6

206.3

7 (9's complement of 793.6) = 999.9 −
793.6
206.3

⇒ Perform the following decimal subtractions using 10's complement in 8421 BCD code.

1. 85 − 26
3. 206.4 − 507.6

2. 342.7 − 108.9

1. 85 − 26

BCD of 85 = 10000101

9's complement of −26 = 99 −
26
73

10's complement of 73 = 73 +
1
74

BCD of 74 = 01110100

85 − 26 ⇒ 10000101 +
01110100
Invalid should add 6 ← 1111,1001
9

∴ 1111 1001 +
0110 0000
discard carry ← ①0101,1001 ⇒ +59

85 −
26
59

2) 342.7 − 108.9

BCD of 342.7 = 001101000010.0111

9's complement of −108.9 = 999.9 −
108.9
891.0

10's complement of 891 = 891 +
1
892

= 891.0 +
1
891.1

BCD of 891.1 = 100010010001.0001

342.7 − 108.9 = 001101000010.0111 +
100010010001.0001
1011,1101,0011.1000
Invalid should add 6 ↓ ↓ 3 8

∴ 101111010011.1000 +
011001100000.0000
discard carry ← ①0010,0011,0011.1000 ⇒ 233.8
2 3 3 .8

342.7 −
108.9
133.8

3. 206.4 − 507.6

BCD of 206.4 = 0010 0000 0110 · 0100

9's complement of −507.6 = 999.9 −
507.6
492.3

10's complement of 492.3 = 492.3 +
1
492.4

BCD of 492.4 = 0100 1001 0010 · 0100

206.4 − 507.6 = 0010 0000 0110 · 0100 +
0100 1001 0010 · 0100
0110 1001 1000 · 1000
6 9 8 · 8

9's complement of 698.8 = 999.9 −
698.8
301.1

10's complement of 301.1 = 301.1 +
1
301.2

Answer = −(301.2)

IMP 4. Convert the decimal numbers 596 and 386 into BCD and do the addition and subtraction operations in BCD arithmetic

5. Subtract 366 from 170 in BCD using 10's complement addition

6. Perform the following decimal operations in 8421 BCD code.
   i) $(518)_{10} + (488)_{10}$
   ii) $(518)_{10} - (488)_{10}$

7. Add the following BCD numbers.
   i) 01100111 + 01010011
   ii) 01111000 + 00100011

78 +
23
101

Floating Point Number Representation

Most common representation today for real numbers is IEEE 754 format. There are 3 basic components in a floating point representation of a number.

1. Sign (s)
   0 → +ve number
   1 → −ve number

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2) <u>Biased Exponents</u> (E)

1. Exponent field needs to represent both
and -ve exponents
2. A bias is added to the actual exponent in
to get the stored exponent.
3. Bias is determined by $2^{k-1}-1$, k - no. of bits in exp field

3) Normalised Mantissa(~~exponent~~ fraction or significant) [F]

1) Mantissa is part of a number in scient notation or a floating point number, consisting of its significant digits.
2) A normalised mantissa is a ~~one with only~~ one with only a one 1 to the left of decimal (binary) p

Homework

4) 1 596+386 ⇒ 010110010110 +
001110000110
1001|0001|1100
9 01 → Invalid should add 6

~~100100001100 +~~
~~0110~~
~~100100010010~~
~~9 1 2~~

100100011100 +
0110
100100100010
9 2 2

596 +
366
982

1011 -
1100

2. 596 - 386 ⇒ 010110010110 −
001110000110
001000010000
2 1 0

596 −
386
210

5) 170−366

9's complement of -366 = 999 −
366
633

10's complement of 633 = 633 +
1
634

366 −
170
196

BCD of 634 = 011000110100 +
BCD of 170 = 000101110000
0111|1010|0100 +
000001100000
1000|0000|0100 ⇒ 803

-(9's complement of 803) = 999 -

803

-196

6) $(518)_{10} + (488)_{10} = (1000)_{10}$

BCD of 518 = 0101 0001 1000 +

BCD of 488 = 0100 1000 1000

1001 1010 0000 → Invalid + Shall

0110

1010 0000 0000

0110

10 0000 0000 0000 ⇒ 1000

$(518)_{10} - (488)_{10}$

= 0101 0001 1000 -

0100 1000 1000

0000 1001 0000

518 -

488

038

7) i) 0110 01111 +

0010 10011

0100 1010 0010

2 2 2

47 +

52

99

ii) 01111000 +

00100011

10011011

0110

10100001

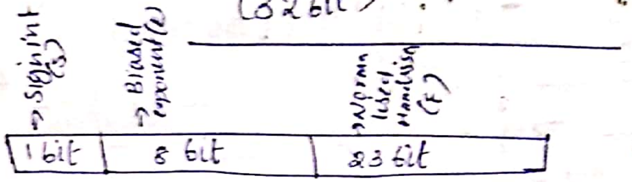
10100001

0110

10000000

101

Single Precision Floating Point Representation (32 bit)



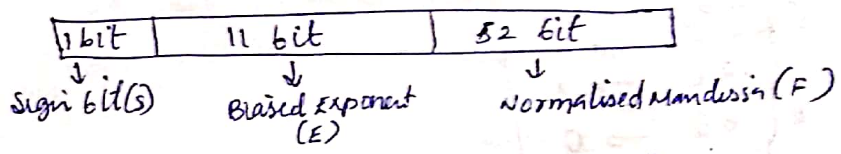
Bias

$2^{k-1} - 1 = 2^7 - 1$

$= 127$

where k = 8

Double precision floating point Representation (64 bit)



Bias

$2^{k-1} - 1 = 2^{10} - 1 = 1023$

k = 11

⇒ 1. Convert the decimal no: 85.125 into single precision floating point binary number

S E

0 10000101 . 01010100100000000000000

2 | 85
2 | 42 → 1
2 | 21 → 0
2 | 10 → 1
2 | 5 → 0
2 | 2 → 1
1 → 0

⇒ Convert the decimal no: 3.248×10^4 into single precision floating point representation

1010101.001

1.010101001×2^6

Ans 1) 3.248×10^4 = 32480

= 111111011100000

= $1·11111011100000 \times 2^{14}$

2 | 32480
2 | 16240 → 0
2 | 8120 → 0
2 | 4060 → 0
2 | 2030 → 0
2 | 1015 → 0
2 | 507 → 1
2 | 253 → 1
2 | 126 → 1
2 | 63 → 0
2 | 31 → 1
2 | 15 → 1
2 | 7 → 1
2 | 3 → 1
1 → 1

2) 85.125

2 | 85
2 | 42 → 1
2 | 21 → 0
2 | 10 → 1
2 | 5 → 0
2 | 2 → 1
1 → 0

0.125 × 2 = 0.25 → 0
0.25 × 2 = 0.50 → 0
0.50 × 2 = 1.00 → 1

Ans:

$85.125 = (1010101.001)_2$

$= 1·010101001 \times 2^6$ → NM

sign bit (s) = 0

Biased Exponent (E)

Bias = 127

$2^{k-1}-1 = 2^7-1$
$=127$

E = Bias + Exponent = 127 + 6
= 133

2 | 133
2 | 66 → 1
2 | 33 → 0
2 | 16 → 1
2 | 8 → 0
2 | 4 → 0
2 | 2 → 0
1 → 0

$(133)_{10} = (1000010)_2$ ⇒ If not 8 bit add 0 to get 8 bit

86
170
150

Normalised Mandissa

Orginal normalised mandissa = 010101001

Normalised mandissa in 23 bit

= 01010100100000000000000

⇒ Find the decimal value corresponding to the given floating point no in single precision form

$\overset{s}{1}\,\underbrace{10000010}_{E}\,\overbrace{11110110000000000000000}^{M(F)}$

Sign (s) = 1 , -ve no

Biased exponent (E) = 10000010

= $\underline{\underline{130}}$

$1\times2^7+1\times$
$=128+2$
$=130$

Original exponent = 130 − 127

= $\underline{\underline{3}}$

Mantissa = $(1.1111011)_2$

$1\times2^7+1\times2^6+1\times2^5$
$+\ 1\times2^{-5}+1\times2^{-6}$

= 1.960 9375

Original decimal value = $(-1)^s \left(1 + \text{normalised mantissa}\right) \times 2^{E}$

= $(-1)^1 \times 1.9609375 \times 2^3$

= $\underline{\underline{-15.6875}}$

⇒ $\overbrace{111001000}1010011000000000000000$

Sign bit (s) = 1 , -ve no.

Biased exponent (E) = 11001000.

= 200

$2^7+2^6+2^3$
$=\underline{\underline{200}}$

original exponent = 200 − 127 = 73

~~Original~~
~~Mantissa = 1.1001000~~

Mantissa = 1.0100111

$1\times2^0+1\times2^{-1}+1\times2^{-3}$

= 2.71

2.625

1.